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ALGORITHMS FOR LOCALIZING ROOTS OF A POLYNOMIAL AND THE PISOT VIJAYARAGHAVAN NUMBERS

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Pisot and Vijayaraghavan studied numbers whose *m*th power is nearly an integer in the sense that the discrepancy vanishes as *m* becomes infinite. One plus square root two is an example. Algebraic numbers of this type are characterized as algebraic integers whose conjugate roots each have absolute value less than one. This note develops a test for this property. An algorithm is given which determines whether or not one root of a polynomial has absolute value greater than one and all the other roots have absolute value less than one. If *n* is the degree of the polynomial, this algorithm involves only *n* rational steps.

1. Introduction. In a previous paper [2] the writer formulated a simple algorithm for testing a polynomial to see if all of its roots were in the interior of the unit circle. Polynomials of this type were termed *Schur polynomials*. The importance of such polynomials rests on the fact that a linear difference equation with constant coefficients is stable if and only if the characteristic polynomial is of Schur type. The test given in [2] follows.

Algorithm A_0. *Let f be the polynomial*

$$f(z) = c_0 + c_1 z + c_2 z^2 + \cdots + c_n z^n$$

where $c_n \neq 0$ *and* $n \geqq 1$. *Let Rf be a polynomial of reduced degree defined with* c_j^* *the complex conjugate of* c_j *as*

$$Rf(z) = (c_n^* c_1 - c_0 c_{n-1}^*) + (c_n^* c_2 - c_0 c_{n-2}^*)z + \cdots + (c_n^* c_n - c_0 c_0^*)z^{n-1} .$$

Then f is a Schur polynomial if and only if:
(i) $|c_n| > |c_0|$,
(ii) *Rf is a Schur polynomial.*

In a series of papers John Miller [4, 5] has extended this algorithmic test for Schur polynomials. Miller's tests determine the "type" of a polynomial, that is, the number of roots inside, on, and outside the unit circle.

The present note was prompted by the works of Pisot and Vijayaraghavan about what may be termed *powerful numbers* [1, 6, 7]. A number greater than one is powerful if its *m*th power