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ON GENERALIZED POLARS OF THE PRODUCT OF ABSTRACT HOMOGENEOUS POLYNOMIALS

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Let E denote a vector space over an algebraically closed field K of characteristic zero. Our object is to investigate the location of null-sets of generalized polars of the product of certain given abstract homogeneous polynomials from E to K. Some special aspects of this general problem were studied in the complex plane by Bôcher and Walsh and, later, in vector spaces by Marden. Our present treatment furnishes further generalizations of the theorems of Marden, Bôcher, and Walsh and offers a systemmatic, abstract, and unified approach to their completely independent methods. One of our results, in special setting, relates to the polar of a product and reduces essentially to the author's earlier generalization [Trans. Amer. Math. Soc., 218 (1976), 115-131] of Hörmander's theorem on polars of abstract homogeneous polynomials. We show also that our theorems cannot be further generalized in certain natural directions.

1. Introduction. Let E be a vector space over a field K of characteristic zero. A mapping P from E to K is called [4, pp. 760-763], [7, p. 55], [8], [14] an *abstract homogeneous polynomial* ($a \cdot h \cdot p \cdot$) of degree n if for every $x, y \in E$,

$$p(sx + ty) = \sum_{k=0}^{n} A_k(x, y)s^k t^{n-k} \quad \forall s, t \in K ,$$

where the coefficients $A_k(x, y) \in K$ and are independent of s and t for any given x, y in E. We shall denote by $\boldsymbol{P}_n$ the class of all nth-degree a.h.p.'s from E to K. The nth-*polar of* P is the mapping (see [5, Lemma 1] for its existence and uniqueness) $P(x_1, x_2, \cdots, x_n)$ from E^n to K which is linear in each x_k and symmetric in the set $\{x_k\}$ such that $P(x, x, \cdots, x) = P(x)$ for every x in E. The kth-*polar* of P is then defined by

$$P(x_1, \cdots, x_k, x) = P(x_1, \cdots, x_k, x, \cdots, x) .$$

The *null-set* $Z_P(x, y)$ of P (relative to elements x, y in E) is defined [9, p. 28], [15] by

$$Z_P(x, y) = \{sx + ty \neq 0 \mid s, t \in K;\ P(sx + ty) = 0\} .$$

Now we shall assume throughout that K is an algebraically closed field of characteristic zero. It is known [5] (see also [2, pp. 38-40],