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INVOLUTIONS OF SEIFERT FIBER SPACES

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A Seifert fiber space M is a compact 3-manifold which decomposes into a collection $\mathscr{F}$ of disjoint simple closed curves, called fibers, such that each fiber has a tubular neighborhood which consists of fibers and is a "standard fibered solid torus." We consider the question, given a PL involution h of M, can the fiber structure $\mathscr{F}$ be chosen in such a way that h will be fiber-preserving? We give an affirmative answer for the case when M is orientable, irreducible, and either $\partial M \neq \emptyset$ or M contains an incompressible fibered torus.

THEOREM. *Let h be a PL involution of the orientable, irreducible Seifert fiber space M. If the orbit-surface (Zerlegungsfläche) of the fiber structure is a 2-sphere, assume in addition that there exist at least four exceptional fibers. Then there exists a Seifert fiber structure on M with respect to which h is fiber-preserving.*

This theorem touches on two earier results. In [6] Montesinos considers the following problem. Given any orientable Seifert fiber space M, determine whether M is homeomorphic to a 2-fold cyclic covering of S^3 branched over a link. He shows that all orientable Seifert fiber spaces with orbit-surface either a 2-sphere or a nonorientable surface are such 3-manifolds. For those with an orientable orbit-surface of positive genus, he compiles a list of all those which are 2-fold branched cyclic covering spaces of S^3 with fiber-preserving covering transformations. We can now conclude that this list is complete since it follows from our theorem that all the PL involutions involved as covering transformations can be viewed as fiber-preserving involutions.

In [1] it is shown that if an irreducible, orientable, sufficiently large 3-manifold M is covered by a compact Seifert fiber space then M is either a Seifert fiber space or the union of two twisted line bundles over a closed nonorientable surface. It is not clear whether the union of these two twisted line bundles admits a Seifert fiber structure, but there exists a two-sheeted covering space M' of the union which is a Seifert fiber space. Thus, if one could show that M' always contains an incompressible fibered torus then it would follow that M is a Seifert fiber space.

Let us describe how one goes about constructing a fiber structure which is preserved by an involution. Let h be a PL involution of the Seifert fiber space M. If M is closed we construct a fibered