PACIFIC JOURNAL OF MATHEMATICS
Vol. 74, No. 2, 1978

ON CHARACTERISTIC HYPERSURFACES OF SUBMANIFOLDS IN EUCLIDEAN SPACE

Keti TENENBLAT

The main purpose of this paper is to prove that $M^n \subset E^N$, where $N = n(n+1)/2$, the characteristic $(n-1)$-dimensional submanifolds of M^n are the asymptotic hypersurfaces.

1. Introduction. The concept of a characteristic submanifold of a given solution for a differential system, was introduced by E. Cartan in his theory of partial differential equations ([2], p. 79). Its importance appears in the treatment of the Cauchy problem.

Given an n-dimensional submanifold M^n of the Euclidean space E^N, we can define geometrically the notion of asymptotic submanifolds of M^n. The asymptotic lines have been used extensively for the study of the geometry of a surface in E^3. For higher dimension and codimension some results have been obtained, using the generalized concept [3], [4], [9], [10]. It is well known, that the characteristic curves of a surface in E^3 are the asymptotic lines ([2], p. 143).

In §2 we start with a brief introduction to the Cartan-Kähler theory of differential equations. Then given a Riemannian manifold M^n, we consider the differential ideal, whose integral submanifolds determine local isometries of M^n into E^N, $N = n(n+1)/2$. Next assuming $M^n \subset E^N$, we characterize the $(n-1)$-dimensional characteristic submanifolds of M^n.

In §3, we define the concept of asymptotic submanifolds of $M^n \subset E^N$, prove the main result and obtain a first order partial differential equation whose solutions are the characteristic hypersurfaces of M.

I am grateful to Professor S. S. Chern for helpful conversations.

2. Characteristic submanifold. Let M be an n-dimensional differentiable manifold. We denote by $\Lambda_k(M)$ the vector space of differential k-forms on M and $\Lambda(M) = \sum_{k=0}^{n} \Lambda_k(M)$. A *differential ideal* is an ideal U in $\Lambda(M)$ which is finitely generated, homogeneous (i.e., $U = \sum_{k=0}^{n} U_k$ where $U_k = U \cap \Lambda_k(M)$) are closed under exterior differentiation. We assume that U is a differential ideal which does not contain functions i.e., $U_0 = 0$. A p-dimensional submanifold S of M is said to be an (p-dimensional) *integral submanifold* for U, if $i^*(U) = 0$ i.e., $i^*(U_p) = 0$ where $i: S \to M$ is the inclusion map.

We denote by T_xM the tangent space to M at $x \in M$; $G_x^p(M)$ denotes the Grassman manifold of p-dimensional subspaces of T_xM