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THE K-THEORY OF AN EQUICHARACTERISTIC DISCRETE VALUATION RING INJECTS INTO THE K-THEORY OF ITS FIELD OF QUOTIENTS

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Let A be an equicharacteristic discrete valuation ring with residue class field F and field of quotients K. The purpose of this note to prove that the transfer map $K_n(F) \to K_n(A)$ is zero $(n \geqq 0)$.

By virtue of Quillen's localization sequence for A, this is equivalent to the statement that the map $K_n(A) \to K_n(K)$ is injective. This result has been conjectured by Gersten and proved by him in the case in which F is a finite separable extension of a field contained in A. We establish the general result by using a limit technique to reduce to this special case.

LEMMA. *Let A be a discrete valuation ring with maximal ideal m and residue class field $A/m = F$. Suppose that A contains a field L; suppose further that F' is a finite separable extension of L satisfying $L \subset F' \subset F$. Then there exists a subring A' of A such that:*

(a) *A' is a discrete valuation ring containing L;*

(b) *$A' \subset A$ is local and flat;*

(c) *if we denote by m' the maximal ideal of A, then $m = m'A$;*

(d) *the image of A' in F is F'; (since $m \cap A = m'$, this implies that we may identify the residue class field of A' with F').*

Proof. Let m be generated by the parameter π. Consider first the case in which A contains a field mapping isomorphically onto F'; let us denote this field also by F'. π is easily seen to be algebraically independent of F', so the subring $F'[\pi]$ of A is isomorphic to a polynomial ring in one variable over F', and π generates a maximal ideal m'. Then $A' = F'[\pi]_{m'}$ is a discrete valuation ring. Furthermore, elements of the complement of m' in $F'[\pi]$ are units in A, so $A' \subset A$. A is flat over A' since A' is Dedekind and A is torsion-free as an A'-module; the other conditions are clear.

Now suppose that A does not contain a field mapping isomorphically onto F'. F' is a simple extension of L, say $F' = L(\bar{\alpha})$; let $f \in L[X]$ be the minimal polynomial of $\bar{\alpha}$. Lift $\bar{\alpha}$ to $\alpha \in A$. If we denote by v the valuation on K, then $v(f(\alpha)) > 0$ since $f(\bar{\alpha}) = 0$