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LIE ALGEBRAS, COALGEBRAS AND RATIONAL HOMOTOPY THEORY FOR NILPOTENT SPACES

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This paper establishes that the homotopy category of rational differential graded commutative coalgebras is equivalent to the homotopy category of rational differential graded Lie algebras which have a nilpotent completion as homology. This generalizes a result which Quillen proved in the simply connected case. When combined with Sullivan's work on rational homotopy theory, our result shows that the homotopy category of rational differential graded Lie algebras with nilpotent finite type homology is equivalent to the rational homotopy category of nilpotent topological spaces with finite type rational homology.

Our results include the construction of minimal Lie algebra models for simply connected spaces, and we show that the rational homotopy groups of a simply connected *CW* complex may be calculated from a free Lie algebra generated by the cells with a differential given on generators by the attaching maps.

0. **Introduction.** Sullivan [4] and Bousfield-Gugenheim [2] have demonstrated the equivalence of two categories: (a) the rational homotopy category of nilpotent spaces with rational homology of finite type, and (b) the homotopy category of commutative, associative, differential graded rational algebras with minimal models of finite type. Earlier, Quillen [23] had demonstrated the equivalence of the rational homotopy category of simply connected spaces with two categories (among others): (c) the homotopy category of commutative, associative, differential graded simply connected rational coalgebras, and (d) the homotopy category of differential graded connected rational Lie algebras. In this paper, we combine these two approaches and generalize the above portion of Quillen's work to nilpotent spaces with rational homology of finite type (Proposition 7.3).

Since we are developing a theory for nilpotent spaces, nilpotent Lie algebras play an important role. As a consequence of our work, we get the purely algebraic result that the homotopy category of commutative differential graded coalgebras is equivalent to the homotopy category of differential graded Lie algebras whose homology is a nilpotent completion (Definition 3.4 and Proposition 7.2).

We associate to each nilpotent space X with rational homology of finite type, three types of differential graded models, a minimal algebra M_X, a minimal coalgebra C_X, and a free Lie algebra $\mathscr{L}(C_X)$.