PACIFIC JOURNAL OF MATHEMATICS
Vol. 74, No. 2, 1978

HEREDITARY CROSSED PRODUCT ORDERS

H. MERKLEN

In this paper one deals with crossed product orders Λ of the following form: Let $\mathscr{R}$ be a Dedekind domain with quotient field $\mathscr{F}$ and $\mathscr{E}$ a semisimple, commutative, algebra of finite dimension over $\mathscr{F}$. Let $\mathscr{G}$ be a finite subgroup of the group of automorphisms of $\mathscr{E}$ whose fixed subalgebra is $\mathscr{F}$, and let Λ_0 be an $\mathscr{R}$-order in $\mathscr{E}$, which is $\mathscr{G}$-stable. Then, if $[\mathfrak{f}]$ is an element of the second cohomology group $H^2(\mathscr{G}, U(\Lambda_0))$, our order is $\Lambda = \Delta(\mathfrak{f}, \Lambda_0, \mathscr{G})$. One is interested in the set of all maximal orders in $\mathscr{A} = \Delta(\mathfrak{f}, \mathscr{E}, \mathscr{G})$ which contain Λ and also in all hereditary orders in $\mathscr{A}$ which contain Λ. In particular, one is interested in knowing sufficient conditions for Λ itself to be hereditary. This last question is answered by Theorem 1, and the other, more general question, is succesively reduced to the classical complete case (i.e., when $\mathscr{R}$ is a local complete Dedekind domain and $\mathscr{E}$ is a Galois field extension of $\mathscr{F}$ with group $\mathscr{G}$), to the totally ramified case (i.e., when, furthermore, $\mathscr{E}/\mathscr{F}$ is totally ramified) and, finally, to the wildly ramified case.

1. Introduction. In general, we use in this paper the terminology of [3]. With $\mathscr{R}$ we denote a Dedekind domain whose field of quotients is $\mathscr{F}$. $\mathscr{A}$ will be a separable $\mathscr{F}$-algebra and $\mathscr{E}$ a finite dimensional, semisimple, commutative subalgebra of $\mathscr{A}$. We denote with $\mathscr{G}$ a finite subgroup of the group of automorphisms of $\mathscr{E}$.

Let $\mathscr{B}$ be a commutative ring with identity and $\mathscr{G}$ a finite group of authomorphisms of $\mathscr{B}$. Then the group of units of $\mathscr{B}, U(\mathscr{B})$, is a G-module. Let $\mathfrak{f}$ be a factor system: $[\mathfrak{f}] \in H^2(\mathscr{G}, U(\mathscr{B}))$. Then we define the ring $\Delta(\mathfrak{f}, \mathscr{B}, \mathscr{G})$ as a free $\mathscr{B}$-module with basis $\{\mathfrak{t}_\sigma\}(\sigma \in \mathscr{G})$, for which a multiplication rule is given by means of:

$$(\mathfrak{b}\mathfrak{t}_\sigma)(\mathfrak{c}\mathfrak{t}_\tau) = \mathfrak{b}\mathfrak{c}^\sigma \mathfrak{f}(\sigma, \tau)\mathfrak{t}_{\sigma\tau} \quad (\mathfrak{b}, \mathfrak{c} \in \mathscr{B}; \sigma, \tau \in \mathscr{G})$$

and extended by additivity.

DEFINITION 1. Given $\mathscr{E}$, a finite dimensional, commutative, semisimple $\mathscr{F}$-algebra, and a finite subgroup $\mathscr{G}$ ofthe automorphisms of $\mathscr{E}$ such that the fixed subalgebra of $\mathscr{E}$ under the action of $\mathscr{G}$ is $\mathscr{F}: \mathscr{E}^{\mathscr{G}} = \mathscr{F}$, we say that $\mathscr{A}$ is a crossed product of $\mathscr{G}$ over $\mathscr{E}$ when $\mathscr{A}$ is isomorphic to the $\mathscr{F}$-algebra $\Delta(\mathfrak{f}, \mathscr{E}, \mathscr{G})$, for some factor system $\mathfrak{f}$ corresponding to an element of $H^2(\mathscr{G}, U(\mathscr{E}))$.

DEFINITION 2. We say that the $\mathscr{R}$-order Λ is a crossed product of $\mathscr{G}$ over Λ_0 when: Λ_0 is an $\mathscr{R}$-order in $\mathscr{E}$, when the fixed subalgebra