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SEQUENCES OF BOUNDED SUMMABILITY DOMAINS

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C. Goffman and G. N. Wollan conjectured that the bounded summability field of a regular matrix A is so thin that the union of countably many such sets is not dense in m. G. M. Petersen proved this conjecture. This result is strengthened by showing if A is a noncoercive matrix whose summability field contains all the finite sequences then its bounded summability field is so thin that the union of countably many such sets is not dense in m. An example is given to show that the condition of containing the finite sequences is necessary.

Preliminaries. Let m and c be respectively the Banach spaces of bounded and convergent sequences, $x = \{x_n\}$, of complex numbers with norm $\|x\|_\infty = \sup_n |x_n|$, $B(x, r) = \{z \in m: \|x + z\|_\infty < r\}$. Denote the nth section of x by $P_n(x) = (x_1, \cdots, x_n, 0, 0, \cdots)$. For each infinite matrix A the set of x transformed by A to convergent sequences is called the summability field of A and denoted by c_A. The set of bounded sequences in c_A is called the bounded summability field of A and is denoted by $\mathscr{A}$. A is called conservative if and only if $c_A \supset c$, regular if and only if A is conservative and limits are preserved, coercive if and only if $c_A \supset m$. If $A = (a_{nk})$, then the A transform of x is designated by $Ax = \{(Ax)_n\} = \{\sum_k a_{nk}x_k\}$. A is conservative if and only if $\|A\|_\infty = \sup_n \sum_k |a_{nk}| < \infty$, $a_k = \lim_n a_{nk}$ exists for each k and $\lim_n \sum_k a_{nk}$ exists [5, p. 165]. A is coercive if and only if $\sum_k |a_{nk}|$ converges uniformly in n and a_k exists for each k [5, p. 169]. Define the essential norm of A by $\|A\|_e = \limsup_n \sum_k |a_{nk} - a_k|$ whenever a_k exists for each k. (Note $\|\ \|_e$ is not a true norm, since $\|\ \|_e$ may be infinite.)

Let E^∞ be the set of all finite sequences and N_0 the set of all sequences of 0's and 1's. Using binary expansions there is a natural injective mapping of $(0, 1)$ onto all but a countable subset of N_0.

MAIN RESULTS. C. Goffman and G. N. Wollan conjectured [4] that the bounded summability field of regular A is so thin that the union of countably many such sets is not dense in m. G. M. Petersen proved this conjecture [6]. We strengthen that result and show that in a certain sense our result is best possible.

THEOREM. *Let $\{A_i\}$ be a countable collection of noncoercive matrices with $\mathscr{A}_i \supset E^\infty$, $i = 1, 2, \cdots$, then $\bigcup_{i=1}^\infty \mathscr{A}_i$ is not dense in m.*

We prove the theorem through a series of lemmas. Since we