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COHERENT POLYNOMIAL RINGS OVER REGULAR RINGS OF FINITE INDEX

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It is shown that polynomial rings in finitely or infinitely many central indeterminates, over a regular ring of finite index, are right and left coherent.

In this paper all rings have unity and all ring homomorphisms preserve the unity.

DEFINITION 1. A ring R is:

(i) Regular, if it satisfies the sentence

$$(\forall r)(\exists s)[rsr = r] ;$$

(ii) Of index n, where $n \geqq 1$ is an integer, if for all $m \geqq n$, it satisfies the sentence

$$(\forall r)[r^m = 0 \longrightarrow r^n = 0] ;$$

(iii) Of finite index if it is of index n, for some integer $n \geqq 1$.

DEFINITION 2. A ring R is left coherent if:

(i) $U \cap V$ is a finitely generated left ideal in R, whenever U and V are finitely generated left ideals in R, and

(ii) For each $r \in R$, the left annihilator of r in R is finitely generated, as a left ideal in R.

Right coherence for R is similarly defined.

DEFINITION 3. Let f be an element of and I a finite subset of a polynomial ring $T[X_1, \cdots, X_q]$.
Then:

(i) $\deg(f)$ is the total degree of f,

(ii) $\deg(I) = \operatorname{Sup}\{\deg(f): f \in I\}$, and

(iii) $\langle I \rangle$ denotes the left ideal generated by I.

It is known (cf. [3, Theorem 2.2]) that a ring is left coherent iff each of its finitely generated left ideals is finitely presented. Thus, for certain homological applications, the left coherent rings seem to be a suitable generalization of the left Noetherian rings. In view of the Hilbert basis theorem (which states that $T[X]$ is left Noetherian if T is), this suggests the following conjecture: if R is a left coherent ring, then $R[X]$ is too. Soublin, in [11], disproved this conjecture, even when R is commutative. However he showed