PACIFIC JOURNAL OF MATHEMATICS
Vol. 74, No. 2, 1978

SUPER TRIANGULATIONS

R. H. BING AND MICHAEL STARBIRD

This paper concerns itself with continuous families of linear embeddings of triangulated complexes into E^2. In [2] Cairns showed that if f and g are two linear embeddings of a triangulated complex (C, T) into E^2 so that there is an orientation preserving homeomorphism k of E^2 with $k \circ f = g$, then there is a continuous family of linear embeddings h_t: $(C, T) \rightarrow E^2(t \in [[0, 1])$ so that $h_0 = f$ and $h_1 = g$. In this paper we prove various relative versions of this result when C is an arc, a θ-curve, or a disk.

Introduction. To appreciate where the results in this paper fit into the literature, it is useful to be aware of the following examples which were described in [1, Example 4.1].

EXAMPLE 1. This example is a triangulated 1-complex (C, S) linearly embedded in E^2 consisting of a simple closed curve J with two disjoint spanning arcs in its interior. The complex C is homeomorphic to ⦷. There is a homeomorphism $g: E^2 \rightarrow E^2$ fixed on J such that $f = g|C$ is linear with respect to S but there is no linear isotopy $h_t: (C, S) \rightarrow E^2(t \in [0, 1])$ with $h_0 = id$ and $h_1 = f$ which keeps J fixed.

EXAMPLE 2. Example 1 can be modified by incorporating (C, S) into the 1-skeleton of a triangulated disk (P, T) with boundary J to produce an example of a disk with properties similar to those of (C, S). Namely, the triangulated disk (P, T) is linearly embedded in E^2 and admits a linear homoemorphism k fixed on $\mathrm{Bd}\, P$ for which there is no linear isotopy $h_t: (P, T) \rightarrow E^2(t \in [0, 1])$ with $h_0 = \mathrm{id}$ and $h_1 = k$ which leaves the boundary fixed throughout.

It is known that no such example can be found where P is convex [1, Corollary 4.4] nor could P be star-like if T has no spanning edge [1, Theorem 4.1].

In this paper it is shown (Theorem 2.4) that no 1-complex homeomorphic to a θ-curve can have the properties of Example 1. Then in Theorem 4.2 it is proved that Example 2 can not retain its properties under all subdivisions. In fact each triangulation T of a disk P has a subdivision T' which is a super triangulation of P. A super triangulation T' of a disk P is one which is as flexible as possible. Namely, any linear embedding of $\mathrm{Bd}\, P$ into E^2 extends to a linear embedding of (P, T') and for any two linear homeomorphisms f, g of (P, T') into E^2 with $f|\mathrm{Bd}\, P = g|\mathrm{Bd}\, P$, there is a linear isotopy