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SEVERAL DIMENSIONAL PROPERTIES OF THE SPECTRUM OF A UNIFORM ALGEBRA

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The author has previously introduced a generalized Šilov boundary which seems useful in studying analytic structure of several dimensions in the spectrum of a uniform algebra $\mathfrak{A}$. Related generalizations of $\mathfrak{A}$-convexity, $\mathfrak{A}$-polyhedra, etc. are developed here. Several different but equivalent approaches to these various generalizations are described. The generalized boundaries discussed here are related to the "q-holomorphic functions" of the author, and to $\mathfrak{A}$-holomorphic convexity.

The generalized Šilov boundary was introduced by the author [2] to study multi-dimensional analytic structure in the spectrum of a uniform algebra. Related but more extensive applications of this boundary were developed by Sibony [13]. Kramm [10] has utilized this boundary to help obtain a characterization of Stein algebras. The definition of the Šilov boundary of order q in [2] was motivated by consideration of $\mathfrak{A}$-varieties of codimension q in the spectrum of $\mathfrak{A}$.

Here we show how extending $\mathfrak{A}$ by the conjugates of q functions from $\mathfrak{A}$, decomposing the spectrum of $\mathfrak{A}$ into $q+1$ pieces, or generalizing the idea of an $\mathfrak{A}$-polyhedron all lead to the same circle of ideas as the qth order boundary. We also relate this boundary to "q-holomorphic" functions. (In [3], [4] the author defined a function f to be q-holomorphic if $\bar{\partial} f \wedge (\partial\bar{\partial} f)^q = 0$, and developed some elementary properties of such functions.) Finally, we establish a connection between the first order boundary and the $\mathfrak{A}$-holomorphic convexity studied by Rickart [11].

We refer the reader to Stout's book, [14], for notation, terminology, and basic results concerning function algebras and uniform algebras.

1. Generalized boundaries and extension algebras.

Let A be a function algebra on the compact Hausdorff space X (although the results of this section also apply if X is locally compact). Let $\partial_0 A$ denote the usual Šilov boundary for A. For a subset S of A let $\# S$ denote the cardinality of S and let

$$V(S) = \{x \in X \mid \forall f \in S, f(x) = 0\} .$$

If K is a closed subset of X define the restriction algebra