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SPECTRAL SYNTHESIS IN SOME SPACES OF BOUNDED CONTINUOUS FUNCTIONS

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Spectral synthesis in the topology of bounded uniform convergence on compact sets is proved for some spaces of bounded continuous functions on the real line R. These spaces include among others the space of continuous functions of bounded variation on R and the space of bounded functions on the real line which are linear combinations of convex functions which satisfy Lipshitz condition of order one uniformly on R.

0. **Introduction.** In the extensive and varied literature which appeared on the subject of spectral synthesis in $L^\infty(R)$, mainly after Malliavin's disproof of spectral synthesis in general [7] the main emphasis was stressed on the study of sets which obey and sets which disobey spectral synthesis. On the other hand, relatively little attention was paid to the investigation of classes of $L^\infty(R)$ functions which admit spectral synthesis. A discussion of the main results which exist in this direction can be found in [1].

The purpose of this paper is to prove for some classes of bounded continuous functions on the real line, that spectral synthesis holds in the topology of bounded uniform convergence on compact sets.

In order to state our main result we need first some notations. For every positive integer n let Δ^n denote the nth difference operator defined inductively on two sided sequences $(a_k)_{k=-\infty}^{\infty}$ by: $\Delta^1 a_k = a_k - a_{k-1}$, $k = 0, \pm 1, \pm 2, \cdots$, and for $n \geqq 1$, $\Delta^n a_k = \Delta^1(\Delta^{n-1} a_k)$, $k = 0, \pm 1, \pm 2, \cdots$. The convention $\Delta^0 a_k = a_k$, $k = 0, \pm 1, \pm 2, \cdots$, will also be adopted.

Our main result is the following:

THEOREM 1. *Let ϕ be a bounded continuous function on R such that for some nonnegative integer n and some real number $1 \leqq p \leqq 2$, the condition*

$$\sup_N \overline{\lim_{h\to 0+}} \sum_{|k|\leqq N/h} \frac{|\Delta^n \phi(kh)|^p}{h^{np-1}} < \infty \tag{1.1}$$

is satisfied. Then ϕ admits spectral synthesis in the topology of bounded uniform convergence on compact sets.

Since the continuous functions on R which satisfy condition (1.1) for $n = p = 1$ are exactly the continuous functions of bounded variation on R, it follows from this theorem that in the conclusion of Theorem 1 in [1] the w^*-topology can be replaced by the stronger