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ON THE RADON-NIKODYM PROPERTY IN A CLASS OF LOCALLY CONVEX SPACES

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In an earlier paper we studied the Radon-Nikodym property (RNP) for Fréchet spaces. D. Gilliam continued the study by examining the RNP for locally convex spaces with the strict Mackey convergence property. The aim of this paper is to take one more step by studying the RNP for the class of locally convex spaces in which every bounded subset is metrizable. Although this class strictly includes the class of spaces with the strict Mackey convergence property, our goal is not a generalization for the sake of generalization. Indeed, we shall prove a theorem that reduces the study of the RNP for this class of spaces directly to the study of the RNP for Banach spaces. This will provide a quick and simultaneous extension of many of the basic Radon-Nikodym theorems in Banach spaces to this class of locally convex spaces. We hope that our technique will eliminate some of the mystery that seems to surround the RNP for locally convex spaces.

1. Definitions and preliminaries. Throughout this paper (E, τ) will always be a quasi-complete locally convex Hausdorff space in which every bounded subset is metrizable and τ will denote its topology.

Let (T, Σ, P) be a probability space and $m: \Sigma \to E$ be a vector measure. For every continuous semi-norm q on E, the q-variation of m over X in Σ is defined to be

$$|m|_q(X) = \sup\left\{\sum_{i=1}^{n} q(m(X_i)); \{X_i\}_{i=1}^{n} \text{ disjoint, } X_i \subset X \text{ and } X_i \in \Sigma \text{ for } 1 \leqq i \leqq n\right\}.$$

The function $|m|_q$ is an extended real-valued measure. The vector measure m is said to be of bounded variation if $|m|_q(T) < +\infty$ for every continuous semi-norm q on E. Also m is said to be P-continuous (denoted $m \ll P$) if $m(X) = 0$, whenever $P(X) = 0$ and $X \in \Sigma$. It is clear that $m \ll P$ if and only if for every continuous semi-norm q on E we have $|m|_q \ll P$. The set

$$\mathrm{Am}(\Sigma) = \left\{\frac{m(X)}{P(X)}; X \in \Sigma, P(X) > 0\right\}$$

is called the *P-average range of m.*