PACIFIC JOURNAL OF MATHEMATICS
Vol. 75, No 1, 1978

PSEUDO-VALUATION DOMAINS

JOHN R. HEDSTROM AND EVAN G. HOUSTON

A domain R is called a pseudo-valuation domain if, whenever a prime ideal P contains the product xy of two elements of the quotient field of R then $x \in P$ or $y \in P$. It is shown that a pseudo-valuation domain which is not a valuation domain is a quasi-local domain (R, M) such that $V = M^{-1}$ is a valuation overring with maximal ideal M. The authors further show that the nonprincipal divisorial ideals of R coincide with the nonzero ideals of V. These ideas are then applied to the case of a Noetherian pseudo-valuation domain R. Such a domain R is shown to have all its nonzero ideals divisorial if and only if each ideal is two-generated. Examples include valuation rings, certain $D + M$ constructions, and certain rings of algebraic integers.

Introduction. The purpose of this paper is to study *pseudo-valuation domains*, a class of rings closely related to valuation rings. We define a *pseudo-valuation domain* to be a domain R in which every prime ideal P has the property that whenever a product of two elements of the quotient field of R lies in P then one of the given elements is in P. One shows easily that valuation rings are pseudo-valuation domains (Prop. 2.1). In the first section of the paper, several characterizations of pseudo-valuation domains are given. For example, a quasi-local domain (R, M) is a pseudo-valuation domain if and only if $x^{-1}M \subset M$ whenever x is an element of the quotient field of R, $x \notin R$ (Th. 1.4).

The name "pseudo-valuation domain" is justified in the second section, first by showing that these rings share many properties with valuation rings. More important is the characterization of a pseudo-valuation domain which is not a valuation domain as a quasi-local domain (R, M) with the property that $V = M^{-1}$ is a valuation overring with maximal ideal M. The second section is concluded with a study of the relationship between the ideals of R and the ideals of V; for example, the set of nonzero ideals of V and the set of nonprincipal, divisorial ideals of R are shown to be one and the same (Cor. 2.15).

In the final section, the authors study Noetherian pseudo-valuation domains. Such rings have Krull dimension $\leqq 1$. Also, a Noetherian pseudo-valuation domain has the 2-generator property if and only if every nonzero ideal is divisorial (Th. 3.5).

Besides valuation rings, two other classes of examples of pseudo-