UNIQUENESS OF LINEAR BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL SYSTEMS

H. Gingold

A unified approach to "Uniqueness of Boundary Value Problems for Linear and Nonlinear Differential Systems" is expounded. New results are obtained and old one's become transparent.

1. Introduction. In this paper we expound a unified approach for determining whether a linear boundary value problem of a non-linear differential system has a unique solution. The advantages of our method are:

(i) It provides a unified approach to linear boundary value problems of a general type.

(ii) It is applicable to linear and non-linear differential systems.

(iii) It provides a tool for investigation of non-linear boundary value problems for non-linear differential systems.

(iv) It makes many old results transparent.

An application of our standpoint to two-point boundary value problems of second order differential systems and equations was demonstrated in Gingold [16].

We will not treat in this paper the uniqueness problem of an n-th order differential equation that includes the De la Vallée-Poussin boundary value problem though our method aplies to that problem as well.

We observe from Gingold [16], that many boundary value problems for n-th order differential equations exhibit "some sort of singularity." Besides this, the importance of the uniqueness of a De la Vallée-Poussin problem (e.g. see Bessmertnykh [6] for an existence theorem, Levin [34–38] for uniqueness) justifies a special discussion. This will be carried out in Gingold and Gustafson [17].

In §2 we introduce assumptions and notations.

In §3 we write a necessary and sufficient condition for the boundary value problems of linear differential systems to possess a unique solution. We also point out how the domain of uniqueness of the boundary value problem may be extended.

In §4 we modify the results of §3 to nonlinear differential systems.

In §5 we illuminate Kim's result [28], and show how nonuniqueness on the boundary ∂D of a simply connected domain D, is related to uniqueness of the boundary value problem in D.

In §6 we discuss sufficient conditions for the uniqueness of a special