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# ANALYTIC DISCS IN THE MAXIMAL IDEAL SPACE OF $M(G)$

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**Let $M(G)$ denote the convolution algebra of finite regular Borel measures on a locally compact abelian group $G$, and let $\Delta$ denote the maximal ideal space of $M(G)$. It is well-known that on certain subsets of $\Delta$ the Gelfand transforms $\mu^\wedge$ of members $\mu$ of $M(G)$ behave like holomorphic functions. The simplest way to exhibit this is to use Taylor's description of $\Delta$ as the semigroup of all continuous semicharacters of a compact semigroup $S$ — the structure semigroup of $M(G)$ (see [10]). If $f \in \Delta(= S^\wedge)$ and $f(s) \geqq 0$ for all $s \in S$, then $f^z \in \Delta$ for $\mathrm{Re}(z) > 0$. Thus, provided $f^2 \neq f$, there is an analytic disc around $f$ in the sense that $\mu^\wedge(f^z)$ is holomorphic on $\mathrm{Re}(z) > 0$ for all $\mu \in M(G)$. Using this fact, Taylor (*loc. cit.*) has shown that if $f$ is a strong boundary point of $M(G)$, then $|f|^2 = |f|$.**

We have already shown ([2]) that there is a point derivation at the idempotent $h$ which corresponds to the direct sum decomposition of $M(G)$ into discrete and continuous measures. It was also possible to prove that this point derivation is continuous in the spectral radius norm so that we were able to deduce that $h$ is not a strong boundary point. Here we strengthen the main result of that earlier paper to show that there is an analytic disc around $h$, and that this disc remains analytic for the completion of $M(G)$ in the spectral radius norm.

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In fact, our methods here are in some ways more straightforward than those we used in that paper, and can be extended to encompass the case when $h$ is an idempotent corresponding to the direct sum decomposition of $M(G)$ induced by a single generator symmetric Raikov system.

The proofs rely heavily on refinements and modifications of techniques given by Williamson in [13] and Varopoulos in [11] in connection with independent subsets of locally compact abelian groups. Indeed our results in §3 are of interest in producing yet another direct sum decomposition of $M(G)$ associated with an independent set. This one lies between the Raikov construction and that of Varopoulos (*loc. cit.*).

In §2 we prove the existence of the disc subject to having a certain decomposition of the measure algebra. Later sections are devoted to the proof of the existence of such a decomposition.