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SETS IN R^d HAVING $(d-2)$-DIMENSIONAL KERNELS

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Let S be a d-dimensional set, $d \geqq 2$, and assume that for every $(d+1)$-member subset T of S, there corresponds a $(d-2)$-dimensional convex set $K_T \subseteq S$ such that every point of T sees K_T via S and $(\operatorname{aff} K_T) \cap S = K_T$. Furthermore, assume that when T is affinely independent, then K_T is the kernel of T relative to S. Then S is starshaped and the kernel of S is $(d-2)$-dimensional.

1. Introduction. Let S be a subset of R^d, $d \geqq 2$. For points x, y in S, we say *x sees y via S* if and only if the corresponding segment $[x, y]$ lies in S. Similarly, for $T \subseteq S$, we say *x sees T* (and *T sees x*) *via S* if and only if x sees each point of T via S. The set of points in S seen by T is called the *kernel of T relative to S* and is denoted $\ker_S T$. Finally, if $\ker_S S \equiv \ker S$ is not empty, then S is said to be *starshaped.*

An interesting problem is that of determining necessary and sufficient conditions for S to be a starshaped set whose kernel is k-dimensional, $0 \leqq k \leqq d$. Several papers have considered this question (Hare and Kenelly **[2]**, Kenelly, Hare, et al. **[3]**, Toranzos **[4]**), and Foland and Marr **[1]** have proved that a set S will have a zero-dimensional kernel provided S contains a noncollinear triple and every three noncollinear members of S see via S a unique common point. Hence the purpose of this paper is to obtain an analogue of these results for subsets of R^d whose kernel is $(d-2)$-dimensional.

The following familiar terminology will be used. Throughout the paper, conv S, aff S, cl S, bdry S, relint S, and ker S will denote the convex hull, affine hull, closure, boundary, relative interior, and kernel, respectively, of the set S. The cone of x over S, defined to be the union of all rays emanating from x through points of S, will be denoted cone(x, S). Also, if S is convex, dim S will represent the dimension of S.

2. Proof of the theorem.

THEOREM. *Let S be a d-dimensional set, $d \geqq 2$, and assume that for every $(d+1)$-member subset T of S, there corresponds a $(d-2)$-dimensional convex set $K_T \subseteq S$ such that every point of T sees K_T via S and $(\operatorname{aff} K_T) \cap S = K_T$. Furthermore, assume that when T is affinely independent, then K_T is the kernel of T relative to S. Then S is starshaped and the kernel of S is $(d-2)$-dimensional.*