PACIFIC JOURNAL OF MATHEMATICS
Vol. 76, No. 1, 1978

A CLASS OF ALGEBRAIC SURFACES OF GENERAL TYPE CONSTRUCTED FROM QUATERNION ALGEBRAS

IRA H. SHAVEL

This paper is concerned with a class of algebraic surfaces of general type constructed from indefinite division quaternion algebras whose centers are totally real number fields. These surfaces are quotients of the product of two upper half planes by Fuchsian groups obtained from the unit groups of maximal orders of such algebras. In the case where the field is real quadratic, we give smoothness conditions for the resulting surfaces and list all smooth surfaces of geometric genus 0. Finally, we give a lower bound for the torsion part of $H^2(\boldsymbol{Z})$.

0. Introduction. From the unit group of a maximal order in a suitable quaternion algebra A over a totally real number field k, one can construct certain Fuchsian groups Γ which can be identified with discrete subgroups of $GL_2^+(\boldsymbol{R})^n$. Γ acts via fractional linear transformation on the product of n copies of the upper half plane to yield a quotient which is known to be a projective algebraic variety. If one takes A to be the total matrix algebra $M_2(k)$, then one obtains the Hilbert modular group of k and the corresponding Hilbert modular variety.

In [4] Hirzebruch studied Hilbert modular surfaces as algebro-geometric and number theoretic objects. The present investigation is primarily geometric, and is concerned with the case where A is division. Unlike Hilbert modular varieties, if A is division the varieties are automatically compact. This avoids the necessity to first compactify and then resolve the resulting cusp singularities.

By a surface we mean a nonsingular, two-dimensional projective algebraic variety. The present surfaces are of general type and have irregularity 0. Those of geometric genus 0 have $c_1^2 = 8$, which distinguishes them topologically from previously known geometric genus 0 general type surfaces which all had $c_1^2 \leqq 3$.

In §1 we describe the basic objects, and in §§2 and 3 we determine the numerical invariants of the surfaces. Necessary and sufficient conditions for smoothness are given in §4. In §5 we give a lower bound for the torsion part of $H^2(\boldsymbol{Z})$, and in the final section we list all examples of geometric genus 0 surfaces of this type arising from real quadratic fields.

This paper is in part based on the author's doctoral dissertation submitted to the State University of New York at Stony Brook. I