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POSITIVE OPERATORS AND THE ERGODIC THEOREM

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Let T be a positive linear operator on $L_1(X, \mathscr{F}, \mu)$ satisfying $\sup_n \|(1/n) \sum_{i=0}^{n-1} T^i\|_1 < \infty$, where $(X, \mathscr{F}, \mu)$ is a finite measure space. It will be proved that the two following conditions are equivalent: (I) For every f in $L_\infty(X, \mathscr{F}, \mu)$ the Cesàro averages of $T^{*n}f$ converge almost everywhere on X. (II) For every f in $L_1(X, \mathscr{F}, \mu)$ the Cesàro averages of $T^n f$ converge in the norm topology of $L_1(X, \mathscr{F}, \mu)$. As an application of the result, a simple proof of a recent individual ergodic theorem of the author is given.

Let $(X, \mathscr{F}, \mu)$ be a finite measure space and T a positive linear operator on $L_1(X, \mathscr{F}, \mu)$. If T is a contraction, then we denote by C and D the conservative and dissipative parts of T, respectively (cf. Foguel [4]). In [5] Helmberg proved that if T is a contraction then the two following conditions are equivalent: (I) For every $f \in L_\infty(X, \mathscr{F}, \mu)$ the Cesàro averages

$$\frac{1}{n}\sum_{i=0}^{n-1} T^{*i}f$$

converge a.e. on X. (II) $\lim_n T^{*n}1_D = 0$ a.e. on X and there exists a function $0 \leqq u \in L_1(X, \mathscr{F}, \mu)$ satisfying $Tu = u$ and $\{u > 0\} = C$. It is easily seen that condition (II) is equivalent to each of the following conditions. (III) For every $u \in L_1(X, \mathscr{F}, \mu)$ the Cesàro averages

$$\frac{1}{n}\sum_{i=0}^{n-1} T^{i}u$$

converge in the norm topology of $L_1(X, \mathscr{F}, \mu)$. (IV) For every $A \in \mathscr{F}$ the Cesàro averages

$$\frac{1}{n}\sum_{i=0}^{n-1} \int T^{*i}1_A d\mu$$

converge. (Cf. Lin and Sine [6].)

The main purpose of this paper is to prove that the equivalence of conditions (I), (III), and (IV) holds, even if T is not a contraction but satisfies $\sup_n \|(1/n) \sum_{i=0}^{n-1} T^i\|_1 < \infty$. That is, we shall prove the

THEOREM 1. *Let $(X, \mathscr{F}, \mu)$ be a finite measure space and T a positive linear operator on $L_1(X, \mathscr{F}, \mu)$ satisfying $\sup_n \|(1/n) \sum_{i=0}^{n-1} T^i\|_1 < \infty$. Then the three following conditions are equivalent:*