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MEIER TYPE THEOREMS FOR GENERAL BOUNDARY APPROACH AND σ-POROUS EXCEPTIONAL SETS

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In this paper we are concerned with determining under what conditions equality is obtained between two different cluster sets of a function f at a point on the boundary of its domain. Specifically for functions defined in the unit disc D in the complex plane taking values in the extended plane we show that the generalized angle cluster set equals the generalized outer angular cluster set at all points of the boundary of D except possibly for a σ-porous set. The definition of both generalized cluster sets includes the usual Stolz angle definition but this result generalizes the known results. In addition the proof is shorter than proofs of less general results.

If f is required to be meromorphic in D then an application of the principal result gives a decomposition of the boundary of D into a set of generalized normal points of f, a set of generalized Picard points of f, and a σ-porous set. The third result gives a different decomposition into generalized Plessner points; generalized pre-Meier points and a σ-porous set. Again these results generalize known results.

The notion of porosity was introduced in 1967 by E. P. Dolženko. While he defined porous sets in higher dimensions as well we shall limit our considerations to C: $|z| = 1$. Porous sets have zero Lebesgue measure and are of first Baire category and in addition isolate properties essential for certain cluster set considerations. In turn this allows generalizations of results of Meier [10] and others. In a series of papers Yoshida [14-20] extended results of Meier and others by using the notion of porosity. Yoshida's fundamental lemma [15, Lemma 1] has a very complicated statement and proof. In this paper we expand this lemma; offer a succinct proof which hopefully illuminates better the character of the results; and apply it to obtain generalizations of some of Yoshida's results as well as those of Dragosh ([6], [7]), Colwell [4] and Yanigahara [13].

2. Definitions and notation. Let D be the open unit disc and $P \subseteqq C$. For each $e^{i\theta} \in C$, let $\gamma(\theta, \varepsilon, P)$ be the length of the largest subarc of the arc $(e^{i(\theta-\varepsilon)}, e^{i(\theta+\varepsilon)})$ which does not meet P. If no such arc exists define $\gamma(\theta, \varepsilon, P) = 0$. According to Dolženko [5], P is *porous* at $e^{i\theta}$ if