PACIFIC JOURNAL OF MATHEMATICS
Vol. 76, No. 1, 1978

ONE-SIDED HEEGAARD SPLITTINGS OF 3-MANIFOLDS

J. H. RUBINSTEIN

For a large class of closed orientable 3-manifolds, we define a new decomposition method which uses embedded one-sided surfaces and is analogous to Heegaard splittings. The technique is most useful for studying some "small" 3-manifolds (i.e., which have finite fundamental group or are not sufficiently large). We give several general criteria for existence of these splittings and some results on nonorientable surfaces in lens spaces. Also stable equivalence (as for Heegaard splittings) and a result of Waldhausen's are shown to carry over to the one-sided case.

0. Introduction. In [7] an example is given showing that the loop theorem is not valid for one-sided surfaces in 3-manifolds. For this reason, such surfaces are difficult to handle and have not been the object of much work. We would like to present a new approach based on the following:

DEFINITION. Let M be a closed orientable 3-manifold. A pair (M, K) is called a *one-sided Heegaard splitting* if K is a closed nonorientable surface embedded in M such that $M - K$ is an open handlebody.

REMARKS. (1) If (M, K) is a one-sided Heegaard splitting and if $N(K)$ is a small closed regular neighborhood of K, then $N(K)$ is homeomorphic to a twisted line-bundle over K. Also $M - \operatorname{int} N(K)$ is a handlebody which we denote by Y.

(2) There is a double cover $p: \tilde{M} \to M$ naturally associated with a one-sided Heegaard splitting (M, K). It is the covering of M corresponding to the subgroup $i_*\pi_1(M - K)$ of $\pi_1(M)$, where i is the inclusion. The surface $p^{-1}(K)$, which we denote by $\tilde{K}$, is the orientable double cover of K and $(\tilde{M}, \tilde{K})$ is a Heegaard decomposition (i.e., the closures of the components of $\tilde{M} - \tilde{K}$ are handlebodies). If $g: \tilde{M} \to \tilde{M}$ is the covering transformation for p then $\tilde{K}$ is g-invariant and g interchanges the components of $\tilde{M} - \tilde{K}$.

We work in the PL category and let M be a closed orientable 3-manifold throughout. In § 1 it is proved that there are one-sided splittings associated with any nonzero class in $H_2(M, Z_2)$. In § 2 one-sided decompositions are discussed where the surface K is (geometrically) incompressible. This is the most useful setting for the theory (cf. [4] for some results employing this approach).