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REALIZING PARTIAL ORDERINGS BY CLASSES OF CO-SIMPLE SETS

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We show that we can embed any countable partial ordering into a class of co-r.e. bi-dense subsets of the rationals, each subset of a fixed nonzero r.e. Turing degree, under an order induced by recursive similarity transformations. Also, we show that we can embed any countable partial ordering into the co-simple isols under either the order induced by addition of isols or the order induced by recursive injections.

0. Introduction. Let C denote the continuum, Q denote the rationals, and N denote the natural numbers. We let c denote the cardinality of C and $\aleph_0$ denote the cardinality of N. Given two linear orderings H and G, we say (i) H is *embeddable* in G, $H \prec G$, if there is an order preserving map from H into G and (ii) H is *similar* to G if there is an order preserving map from H onto G. H is said to be *bi-dense* in G if $H \subseteqq G$ and both H and $G - H$ are dense in G.

Let π be an effective one-one correspondence between Q and the natural numbers. We shall consider π to be an effective Gödel numbering and thus we will identify an element or subset of Q with its image under π. We let $\leqq$ or $<$ refer to the usual ordering on N and ⩽ or ⧀ refer to the usual ordering on Q. Given $\alpha, \beta \subseteqq Q$, we say α is *recursively embeddable* in β, $\alpha \prec_c \beta$, if there is a partial recursive function φ such that $\alpha \subseteqq \delta\varphi$, the domain of φ, and the restriction of φ to α, $\varphi \restriction \alpha$, is an order preserving map from α into β.

In [5], Hay, Manaster, and Rosenstein show that complements of recursively enumerable bi-dense subsets of Q of any fixed nonzero r.e. degree under $\prec_c$ bear a strong resemblance to bi-dense subsets of C of cardinality c under $\prec$. The main result of this paper answers a question raised by Laver. Based on the results of [5], Laver asked whether or not the following theorem is true.

THEOREM A. *Let β be any recursively enumerable set which is not recursive and let P be any countable partial ordering. Then there is a collection of co-recursively enumerable bi-dense subsets of Q, each Turing equivalent to β, such that, under $\prec_c$, this collection is order isomorphic to P.*

(A set $A \subseteqq N$ is co-recursively enumerable if $N - A$ is recursively enumerable.) In §2 of this paper, we prove Theorem A using methods that Sack's [8] developed to prove that any countable partial ordering