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TANGENT FRAME FIELDS ON SPIN MANIFOLDS

DUANE RANDALL

In this note we prove the following theorems.

THEOREM A. **Let M^n be a spin manifold with $n \equiv 7 \bmod 8$ and $n > 7$. Then M admits at least 8 nonhomotopic tangent 4-frame fields.**

THEOREM B. **Let M^n be a spin manifold with $n \equiv 3 \bmod 8$ and $n > 3$. Suppose that $w_{n-4}M = 0$ and $w_4M \cdot w_{n-5}M = 0$. Then M^n admits a tangent 4-frame field iff**

$$w_{n-3}M = 0 \quad \text{and} \quad \chi_2 M = 0 \ .$$

1. **Introduction.** Here M^n denotes a closed connected smooth manifold of dimension n. A tangent k-frame field on M^n is an ordered set of k linearly independent vector fields on M^n. The classical theorem of Hopf states that M^n possesses a tangent 1-frame field iff the Euler characteristic $\chi M = 0$. A table of necessary and sufficient conditions for tangent 2-frame fields on orientable manifolds appears in [10] while conditions for tangent 3-frame fields are tabulated in and [3]. In particular, Atiyah and Dupont prove in [1] that any orientable manifold M^n with $n \equiv 3 \bmod 4$ admits a tangent 3-frame field. This result is best possible since neither the sphere S^{8i+3} nor $S^3 \times CP^{4i+2}$ admits a tangent 4-frame field.

Recall that an orientable manifold M^n is called a spin manifold if the Stiefel-Whitney class w_2M is trivial. The mod 2 semicharacteristic $\chi_2 M^n$ is defined if $n = 2s + 1$ by

$$\chi_2 M = \left(\sum_{i=0}^{s} \dim H_i(M;\, Z/2)\right) \bmod 2 \ .$$

Let σM denote the signature of M^n whenever n is divisible by 4. Finally δ represents the Bockstein-coboundary operator associated to the exact coefficient sequence $Z \to Z \to Z/2$.

Theorem A is a best possible result for $n \equiv 7 \bmod 16$. In [8, p. 690] Szczarba constructed certain spin manifolds M^n with $n \equiv 3 \bmod 4$ as the quotient spaces of free and differentiable actions of generalized quarternion groups on S^n. The span of these spherical space forms M^n with $n \equiv 7 \bmod 16$ and $n > 7$ is precisely 4 by Theorem 1.1 of [2].

An immediate consequence of Theorem A and the result of Thurston given by [14, Corollary 1] is the following.

COROLLARY. *Let M^n be a spin manifold with $n \equiv 7 \bmod 8$ and*