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CONTINUOUS CANONICAL FORMS FOR MATRICES UNDER UNITARY EQUIVALENCE

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A continuous canonical form for the unitary equivalence of 2×2 complex matrices is constracted and it is proved that for $n \geqq 3$ there does not exist a continuous canonical form for the unitary equivalence of $n \times n$ complex matrices. Both results have applications to the study of singly generated C^*-algebras of type I_n.

1. Introduction. Let M_n be the ring of $n \times n$ complex matrices with its usual topology. We shall call a function $J: M_n \to M_n$ *a canonical form for the relation of unitary equivalence* on M_n if J satisfies:

(i) $J(A)$ is unitarily equivalent to A,

(ii) if B is unitarily equivalent to A, then $J(A) = J(B)$.

In other words, a canonical form for the relation of unitary equivalence is a rule for selecting a unique matrix from each unitary equivalence class.

If the equivalence relation of similarity is considered instead of the relation of unitary equivalence, then the map which sends each matrix to its Jordan form is a well known and reasonably computable example of a canonical form for the relation of similarity. The problem of constructing a canonical form for the relation of unitary equivalence does not have as satisfactory a solution as the Jordan form yields for similarity. There are several reasons for this. First, the problem of finding suitable invariants that determine when two $n \times n$ matrices are unitarily equivalent was solved more recently than the corresponding problem for similarity. A complete set of unitary invariants for $n \times n$ complex matrices was first found by Specht [30], but the number of invariants given was infinite. Specht proved that if W denotes the free multiplicative semi-group generated by the symbols x and y, then two $n \times n$ matrices A and B are unitarily equivalent if and only if $\mathrm{Tr}\,(w(A, A^*)) = \mathrm{Tr}\,(w(B, B^*))$ for all $w(x, y) \in W$, where $\mathrm{Tr}\,(\cdot)$ is the trace function. A finite set of invariants was given in [24, Thm. 2] where for n fixed but arbitrary, a subset of W containing 4^{n^2} elements which forms a complete set of unitary invariants is explicitly constructed. The sharpness of the above bound is not known, but a detailed analysis of the cases $n = 2$ and $n = 3$ in [21] and [23], respectively, shows that considerably fewer invariants are sufficient. The second difficulty encountered in constructing a canonical form for the relation