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ON MASSEY PRODUCTS

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We present two examples: one showing the necessity of the condition that the two triple products must vanish *simultaneously* in order for the quadruple product to be defined; and one showing that the higher order products of a path connected, simply connected space are not completely determined by the differentials in the Eilenberg-Moore spectral sequence of its path-loop fibration.

1. Introduction. This paper is concerned with two examples concerning Massey products. Example I fills a gap in the literature since there are many references (see [2], [1], [3]) to the fact that the two triple products $\langle u, v, w\rangle$ and $\langle v, w, x\rangle$ must vanish *simultaneously* in order for the quadruple product $\langle u, v, w, x\rangle$ to be defined, yet there appears to be no example proving this. Example II gives a path connected, simply connected space which has higher order products which are not determined by the differentials in the Eilenberg-Moore spectral sequence of its path-loop fibration. This is to show that the higher products are a richer source of information about a space than the above-mentioned spectral sequence, and this should be contrasted with J. Peter May's result that matrix Massey products completely determine the differentials in the Eilenberg-Moore spectral sequence (see [4]).

2. Definitions. Because of different conventions in the literature used to define Massey products and to state what it means for the two triple products to vanish simultaneously, we present the following definitions:

Let X be a topological space and R a commutative ring with identity. $H^*(X; R)$ will denote the singular cohomology ring and $C^*(X; R)$ the singular cochain complex. (We could use in these definitions any cochain complex which has an associative product.) If $a \in H^p(X; R)$ or $C^p(X; R)$, we will write $\bar{a} = (-1)^p a$. We first define the triple product.

DEFINITION 1. Let u, v, and w be homogeneous entries from $H^*(X; R)$ of degrees p, q, and r respectively. Choose representative cocycles u', v', and w' for these classes respectively, and assume that $uv = 0$ and $vw = 0$. We may select cochains a^{12} and a^{23} such that

$$\delta(a^{12}) = \overline{u'}v'$$
$$\delta(a^{23}) = \overline{v'}w' .$$