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ON THE EXPANSION IN JOINT GENERALIZED EIGENVECTORS

KLAUS GERO KALB

Let $\mathscr{A}$ be a family commuting selfadjoint of (normal) operators in a complex (not necessarily separable) Hilbert space H. A natural triplet $\phi \subset H \subset \phi'$ is described, such that (1) $\mathscr{A}$ possesses a complete system of joint generalized eigenvectors in ϕ'; (2) the joint generalized point spectrum of $\mathscr{A}$ essentially coincides with the joint spectrum of $\mathscr{A}$; (3) the generalized point spectra, generalized spectra and spectra essentially coincide for all $A \in \mathscr{A}$; (4) the simultaneous diagonalization of $\mathscr{A}$ in H by means of its spectral measure extends to ϕ'. Also the multiplicity of the joint generalized eigenvectors of $\mathscr{A}$ is discussed.

Let ϕ be a locally convex space, which is embedded densely and continiously into H, such that $A\phi \subset \phi$ and $\dot{A} = A|\phi \in \mathscr{L}(\phi)$ for all $A \in \mathscr{A}$. Consider the triplet $\phi \subset H \subset \phi'$. A joint generalized eigenvector of $\mathscr{A}$ with respect to the joint generalized eigenvalue $(\lambda_A)_{A \in \mathscr{A}} \in \prod_{A \in \mathscr{A}} C$ is a continuous linear form $x' \in \phi'$ such that

$$(1.1) \qquad x' \neq 0 \quad \text{and} \quad \dot{A}'x' = \lambda_A \cdot x' \quad \text{for all} \quad A \in \mathscr{A}.$$

The system $\mathfrak{E}$ of all joint generalized eigenvectors of $\mathscr{A}$ is called complete, if $\langle \varphi, e' \rangle = 0$ for all $e' \in \mathfrak{E}$ implies $\varphi = 0$ ($\varphi \in \phi$). For H separable there is a number of conditions on ϕ, under which $\mathfrak{E}$ is complete (cf. e.g., [14], [3]), and there also are effective constructions of ϕ with respect to a given family $\mathscr{A}$ (cf. [13], [14] for $\mathscr{A}$ countable; [15]). The fact that especially in the case of a single normal operator there generally exist many more joint generalized eigenvalues and eigennvectors than necessary (and reasonable in physical applications) has led to recent investigations ([15], [16]; [1]; [2]; [5]; [8], [9]). Let $\sigma_P(\dot{\mathscr{A}}')$ be the joint generalized point spectrum of $\mathscr{A}$ (i.e., the set of all joint generalized eigenvalues of $\mathscr{A}$), let $\sigma(\mathscr{A})$ be the joint spectrum of $\mathscr{A}$ as defined in Gelfand theory (cf. § 2). Let $\mathscr{B}$ be the (commutative) C^*-algebra generated by $\mathscr{A}$ and 1. In the present work we propose the construction of a natural triplet $\phi \subset H \subset \phi'$, by which the following is achieved:

(a) $\sigma_P(\dot{\mathscr{A}}') \subset \overline{\sigma_P(\dot{\mathscr{A}}')} = \sigma(\mathscr{A})$;

(b) $\sigma_P(\dot{B}') \subset \overline{\sigma_P(\dot{B}')} = \sigma(\dot{B}') = \sigma(B)$ for all $B \in \mathscr{B}$;

(c) the simultaneous diagonalization of $\mathscr{B}$ by means of its spectral measure can be transferred to $\dot{\mathscr{B}}'$.