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EXISTENCE OF A STRONG LIFTING COMMUTING WITH A COMPACT GROUP OF TRANSFORMATIONS

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Let G be a locally compact group with left Haar measure γ. The well-known "Theorem LCG" ([10]) states that there is a strong lifting of $M^\infty(G, \gamma)$ commuting with left translations. We will prove partial generalizations of this theorem in case G is *compact*. Thus, let (G, X) be a *free* (left) transformation group with G, X compact such that (I) G is abelian, or (II) G is Lie, or (III) X is a product $G \times Y$. Let ν_0 be a Radon measure on $Y = X/G$, and let μ be the Haar lift of ν_0 We will show that, if ρ_0 is a strong lifting of $M^\infty(Y, \nu_0)$, then there is a strong lifting $M^\infty(X, \mu)$ which extends ρ_0 and commutes with the action of G.

The proof is modeled on the proof of LCG in ([10]), and follows it closely in several places. The main difference is in the present use of the fact that, if (H, X) is a free transformation group with H *Lie*, then (H, X) admits local sections.

DEFINITIONS 1.1. Let X be a compact Hausdorff space. Let $M_+(X)$ denote the set of positive Radon measures on X of norm 1 with the vague topology. For measure theory, we rely on [2], [3], [4]. If $\eta \in M_+(X)$, let $M^\infty(X, \eta)$ be the set of all bounded η-measurable complex functions on X. If $f \in M^\infty(X, \eta)$, let $N_\infty(f)$ denote its essential supremum. Let $L^\infty(X, \eta)$ be the usual set of equivalence classes modulo null functions.

Define $L^p(X, \eta)$ in the usual way; let N_p be its norm $(1 \leqq p < \infty)$. Since X is compact, we can and will assume that

$$L_p(X, \eta) \subset L^r(X, \eta) \quad (1 \leqq r \leqq p \leqq \infty) .$$

DEFINITIONS 1.2. Let W be a topological space, $f: X \to W$ a map. Say f is *η-Lusin-measurable* if there is a countable collection of pairwise disjoint compact sets K_i such that $X \backslash \bigcup_i K_i$ has η-measure zero and $f|_{K_i}$ is continuous $(i \geqq 1)$.

DEFINITIONS, NOTATION 1.3. Let G be a compact Hausdorff topological group. The pair (G, X) is a *free* (left) *transformation group* (t.g.) if there is a jointly continuous map $G \times X \to X: (g, x) \to g \cdot x$ such that, if $g \cdot x = x$ for any $g \in G$ and $x \in X$, then $g = \text{idy}$, the