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THE HOMOTOPY TYPE OF THE SPACE OF MAPS OF A HOMOLOGY 3-SPHERE INTO THE 2-SPHERE

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It is proved that if K is a compact, connected polyhedron such that $H^2(K; \mathbf{Z}) = 0$, then all the components in the space of maps of K into the 2-sphere are homeomorphic. For K a polyhedral homology 3-sphere the common homotopy type of the components is identified and shown to be independent of K.

1. Introduction and statements of results. Let K and X be a pair of compact, connected polyhedra and let $M(K, X)$ denote the space of (continuous) maps of K into X. All mapping spaces will be equipped with the compact-open topology. Corresponding to each homotopy class of maps of K into X there is a (path-) component in $M(K, X)$. For each pair of spaces K and X there arises then a natural classification problem, namely that of dividing the set of components in $M(K, X)$ into homotopy types. The present paper is one in a series of papers, where we search through classical algebraic topology for methods, which are useful in the study of such classification problems.

In [4], information on certain Whitehead products was used to tackle the classification problem for the set of components in the space of maps of the m-sphere S^m into the n-sphere S^n, $m \geqq n \geqq 1$, and complete solutions were obtained in the cases $m = n$ and $m = n + 1$. If the domain in the mapping space is not a suspension, the problem becomes more delicate, since normally, it is then difficult to construct nontrivial maps between the various components. For a mapping space with a manifold as domain it is sometimes possible to solve the classification problem for the components using information about a corresponding mapping space with a sphere as domain. As an example, knowledge of the fundamental group of the various components in $M(S^2, S^2)$ was used in [5] to solve the classification problem for the countable number of components in the space of maps of an orientable closed surface into S^2. In this paper, we shall investigate spaces of maps into the base space of a principal bundle. We will concentrate mainly on spaces of maps into S^2, making use of the fact, that S^2 is the base space in a principal S^1-bundle, namely the classical Hopf fibration $p: S^3 \rightarrow S^2$.

The main result in this paper is the following