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POTENTIAL OPERATORS AND EQUIMEASURABILITY

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W. Rudin proved the following.

THEOREM 1.1. **Assume $0 < p < \infty$, $p \neq 2, 4, 6, \cdots$. Let n be a positive integer. If $f_i \in L_p(\mu)$, $g_i \in L_p(\nu)$ for $1 \leqq i \leqq n$ and**

$$\int_X |1 + z_1 f_1 + \cdots + z_n f_n|^p d\mu = \int_Y |1 + z_1 g_1 + \cdots + z_n g_n|^p d\nu$$

for all $(z_1, \cdots, z_n) \in C^n$, then $(f_1, \cdots, f_n)$ and $(g_1, \cdots, g_n)$ are equimeasurable. Here as usual $L_p(\mu)$ and $L_p(\nu)$ stand for pth power integrable functions defined on finite measure spaces (X, X, μ) and (Y, Y, ν) respectively. $\mathscr{C}$ is the field of complex numbers.

The purpose of this paper is to provide a new setting for Rudin's result by recasting it and its extension to real valued functions into the framework of the theory of potential operators as formulated by K. Yosida.

We begin by outlining the theory of potential operators ([8], [9]). K-I. Sato's 1970 paper [6] contains extensive material on the subject.

Let T_t be a strongly continuous semigroup of linear operators on a Banach space X, satisfying $\sup_t \| T_t \| < +\infty$, with infinitesimal generator $A = s \lim_{t \to 0} (T_t f - f)/t$ where as usual s denotes the strong limit, and resolvent $J_\lambda = (\lambda - A)^{-1}$, $\lambda > 0$. K. Yosida defined the potential operator V as follows:

$$Vf = s \lim_{\lambda \to 0} J_\lambda f \ ,$$

if the strong limit exists for a dense set in X. This is one way to unify the potential operator concept for a large class of Markov processes, which includes Brownian motion, stable processes and of course transient Markov processes.

Motivated by an application to equimeasurability in Section (3), we specialize to potential operators induced by Markov processes. Thus let S be a locally compact, noncompact, Hausdorff space with countable basis. By $C_0(S)$, $C_k(S)$ we denote the spaces of real valued functions which vanish at infinity, and those with compact support respectively. Let T_t be a strongly continuous semigroup of positive linear operators on $C_0(S)$ with $\| T_t \| \leqq 1$. To this semigroup there corresponds a right continuous Markov process $\{X_t\}$ on S with transition $P_t(x, A)$, such that:

$$T_t f(x) = \int P_t(x, dy) f(y) \ .$$