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UNIVERSAL INTERPOLATING SETS AND THE NEVANLINNA-PICK PROPERTY IN BANACH SPACES OF FUNCTIONS

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1. Introduction. Let E be a Banach space of functions on S, $W \subset S$, and let $M(E)$ be the multiplier algebra of E. Consider the restriction space $E \mid W$ as a quotient of E. The space E has the *Nevanlinna-Pick property* relative to W if $M(E \mid W) = M(E) \mid W$ isometrically; E has the *factorization property* relative to W if there exists $u \in M(E)$ such that u is an isometry of $E \mid W$ onto the annihilator of S/W in E. We consider the problem of characterizing those spaces with the Nevanlinna-Pick property.

Theorem 1 solves this problem for suitable sequence spaces. It is shown that the Nevanlinna-Pick property of E is equivalent to a natural factorization property of annihilators in the series space of E. It follows that E has the Nevanlinna-Pick property relative to W whenever $M(E)$ has the factorization property relative to W. A technique is provided in Lemma 6 for applying these sequence space results to general Banach spaces of functions. An identification of the dual of $H^2 \mid W$ yields a proof of the classical Nevanlinna-Pick theorem based solely on the elementary factorization theory of the Hardy spaces. Zero set considerations yield the failure of the Nevanlinna Pick theorem in the Bergman spaces.

Applications are given to universal interpolating set problems in general Banach spaces of functions. Let $l^2(S)$ be the usual Hilbert space of functions on S where S has counting measure. Let H be a Hilbert space of functions on S. A subset W of S is a *universal interpolating set for* H if there exists a multiplier from $H \mid W$ onto $l^2(W)$. We show that W is a universal interpolating set for H if and only if $M(H \mid W) = l^\infty(W)$, the space of bounded functions on W. This result provides a convenient definition of universal interpolating sets for general Banach spaces of functions. It follows that if E and F are Banach spaces of functions on S, $M(E) \subset M(F)$, W is a universal interpolating set for E, and E has the Nevanlinna-Pick property relative to W, then W is a universal interpolating set for F. These results provide generalizations of some theorems of Shapiro and Shields on weighted interpolation in the Hardy space H^2 and the Bergman space A^2.

Finally, it is shown under weak assumptions that universal interpolating sequences always exist for Hilbert spaces of functions but may fail to exist for Banach spaces of functions.