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(*hnp*)-RINGS OVER WHICH EVERY MODULE ADMITS A BASIC SUBMODULE

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The structure of those bounded (*hnp*)-rings over which every module admits a basic submodule, is determined. It is shown that such rings are precisely the block lower triangular matrix rings over $D\backslash M$ where D is a discrete valuation ring with M as its maximal ideal.

In [12], the author generalized some well known results on decomposability of torsion abelian groups to torsion modules over bounded (*hnp*)-rings. Let R be a bounded (*hnp*)-ring and M be a (right) R-module. A submodule N of M is called a *basic* submodule of M if it satisfies the following conditions:

(i) N is decomposable in the sense that it is a direct sum of uniserial modules and finitely generated uniform torsion free modules.

(ii) N is a pure submodule of M.

(iii) M/N is a divisible module.

The following result has been proved by the author (see [9] for details):

THEOREM 1. *Any torsion module M over a bounded* (*hnp*)-*ring has a basic submodule and any two basic submodules of M are isomorphic.*

In general an R-module need not have a basic submodule. However Marubayashi [8, Theorem (3.6)] showed that every module over a g-discrete valuation ring has a basic submodule. In this paper we determine the structure of those bounded (*hnp*)-rings, over which every (right) module admits a basic submodule (Theorems 3 and 4).

As defined by Marubayashi [8, p. 432], a prime, right as well as left principal ideal ring R, such that its Jacobson radical $J(R)$ is the only maximal ideal, and idempotents modules $J(R)$ can be lifted, is called a g-discrete valuation ring; further if $R/J(R)$ is a division ring, then R is called a discrete valuation ring. In view of [8, Lemma (3.1)] and [7, Lemma (2.1)], g-discrete valuation rings are precisely the matrix rings over discrete valuation rings. Modules considered will be unital right modules and the notations and terminology of [12, 13] will be used without comment.

Henceforth in all lemmas, R is a bounded (*hnp*)-ring over which every module admits a basic submodule. Further Q stands for the classical quotient ring of R.