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# CHARACTERIZING THE ORDERS CHANGED BY PROGRAM TRANSLATORS

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**The ways in which translators from one programming system for the recursively enumerable sets to another such programming system can change the orders of the sets being translated are characterized using the computable functions which permute infinitely many initial segments.**

In [3], it is shown (Corollary, p. 194) that every translator from one programming system for the recursively enumerable (r.e.) sets to another such programming system must preserve every order of enumeration of every r.e. set on infinitely many of the programs which enumerate the set in the given order. It was also conjectured there that for every translator, many sets of cardinality greater than one *never* have their order of enumeration changed by the translation of *any* of their programs. In this paper, we show that this conjecture is false, although "nearly" true, and we characterize the orders which can be changed by program translators. Specifically, we show that given any r.e. sequence of effective permutations which permute infinitely many initial segments, we can build a translator which changes every (infinite) order of enumeration by every permutation in this set. On the other hand, if a program enumerates a set sufficiently slowly, then no translation of the program can change the order of enumeration by a permutation which is not of this form. Thus for any translation, many sets (those having only slow enumerations) have *all* of their enumeration orders preserved *modulo* such permutations of their initial segments.

In [3], the vague conjecture that "the only general method of translation is simulation (of the source programs)" is discussed. The results presented here are compatible with that conjecture.

We use without further discussion the notation and the definitions of [3], and we assume some familiarity with the results of that paper.

DEFINITION Let $p$ by a function on the natural numbers, i.e., $p\colon N \xrightarrow[\text{onto}]{1-1} N$. Then $p$ permutes initial segments if there are infinitely many $n$ such that $\{p(i) \mid i < n\} = \{i \mid i < n\}$.

We first show that, in a very strong sense, translations can change orders of enumerations by functions which permute initial segments. Intuitively, if $p$ is such a permutation, and we want to