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SOME RADICAL PROPERTIES OF RINGS WITH $(a, b, c) = (c, a, b)$

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A ring is an s-ring if (for fixed s) A^s is an ideal whenever A is. We show that at least two different definitions for the prime radical are equivalent in s-rings. If R satisfies $(a, b, c) = (c, a, b)$ then R is a 2-ring. In this note we investigate various properties of the prime and nil radicals of R. In addition, if R is a finite dimensional algebra over a field of characteristic $\neq 2$ of 3 we show that the concepts of nil and nilpotent are equivalent.

In [1] Brown and McCoy studied a collection of prime radicals and nil radicals in an arbitrary nonassociative ring. In light of their treatment we will consider these radicals in rings which satisfy the identity

$$(1) \qquad (a, b, c) = (c, a, b) .$$

While these rings may be viewed as an extension of alternative rings, they are in general not even power associative. Examples of (not power associative) rings satisfying (1) appear in [2] and [4].

1. s-rings and the prime radical. Prime radicals for an arbitrary ring R were treated in [1] in the following way. Let $\mathscr{A}$ be the set of all finite nonassociative products of at least two elements from some countable set of indeterminates $x_1, x_2, x_3, \cdots$. Then if $u \in \mathscr{A}$ we call an ideal P u-prime if $u(A_1, A_2, \cdots, A_n) \subseteqq P$ implies some $A_i \subseteqq P$ for ideals $A_1, A_2, \cdots, A_n$. For example if $u = (x_1x_2)x_3$ then P is u-prime if whenever $(A_1A_2)A_3 \subseteqq P$ we have one of the A_i's in P. The u-prime radical R^u is then the intersection of all u-prime ideals in R. It was shown that if u^* is the word having the same association as u, but in only one variable, then $R^u = R^{u^*}$. For example if $u = (x_1x_2)x_3$ then $u^* = (xx)x$, and R^{u^*} is the intersection of ideals P with the property that if $(AA)A \subseteqq P$ for an ideal A, then $A \subseteqq P$.

Another theory of the prime radical was given in [9]. Call a ring R an s-ring if for some fixed positive integer s, A^s is an ideal whenever A is. Call an ideal P prime if $A_1A_2 \cdots A_s \subseteqq P$ implies some $A_i \subseteqq P$ for ideals $A_1, \cdots, A_s$. The prime radical $P(R)$ of an s-ring R is then the intersection of all prime ideals.

In the case of s-rings we see that these approaches are essentially the same: