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THE SCHUR GROUP OF A FIELD OF CHARACTERISTIC ZERO

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We determine when a class in the Schur subgroup $S(K)$ of the Brauer group $B(K)$ of a field K of characteristic zero contains an algebra which is isomorphic to a simple summand A of the group algebra FG for some finite group G, where F is a subfield of K. We then investigate $A \otimes_F K$ which is the direct sum of simple algebras with center K, and determine exactly when these are K-isomorphic. Finally we refine existing examples in the theory of the Schur group, and obtain a decomposition theorem for the related group of algebras with uniformly distributed invariants.

In the introduction to [6] Janusz notes that: For a finite abelian extension K of the rationals Q, $S(K)$ the Schur subgroup of the Brauer group $B(K)$ consists of all classes $[A]$ consisting of an algebra A which is isomorphic to a simple summand of the group algebra QG for some finite group G. Our first result in this paper is that for an arbitrary field K of characteristic zero the above is false.

In Mollin [8-13] we develop the concept of the uniform distribution group $U(K)$ for an algebraic number field K. If ε_{p^a} denotes a primitive p^ath root of unity and ε_{p^a} is the highest p-power root of unity in K we have from Mollin [8] that when p does not divide $|K: Q(\varepsilon_{p^a})|$ then:

$$(*) \qquad U(K)_p = S(K)_p = K \otimes S(Q(\varepsilon_{p^a}))_p$$

where G_p denotes the p-primary part of a group G. In Janusz [7] it is shown that if p does not divide $|Q(\varepsilon_n): K|$ when $Q(\varepsilon_n)$ is the smallest root of unity field containing K then:

$$(**) \qquad S(K)_p = K \otimes S(Q(\varepsilon_{p^a}))_p \,.$$

C. Ford and G. Janusz [5] give for each prime p, examples of fields K for which $(**)$ does not hold. In this paper we present, for each prime p, fields K for which the second equality of $(*)$ does not hold but for which the first equality does hold. Finally, we obtain a decomposition theorem for $U(K)$.

2. **Notation and preliminaries.** Let K be a field of characteristic zero. The Schur group $S(K)$ may be described as consisting of those equivalence classes in $B(K)$ which contain a simple component of the group algebra KG for some finite group G. By