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CENTERS OF REGULAR SELF-INJECTIVE RINGS

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This paper is concerned with calculating centers of regular self-injective rings, particularly those obtained by completions with respect to rank functions, and those obtained as factor rings of other regular self-injective rings. Sufficient conditions are developed under which the completion of a regular ring R has the same center as R. For any regular self-injective ring R of Type I_f, it is shown that the center of any factor ring of R is a factor ring of the center of R. These results are used to distinguish among the simple regular self-injective rings of Type II_f by their possible centers.

All rings in this paper are associative with unit, and all ring maps are assumed to preserve the unit.

1. Introduction. The class of regular, right self-injective rings may be divided into subclasses using the theory of types as in [6, Chapters 5-7]. In particular, any indecomposable, regular, right self-injective ring must be one of Types I_f, I_∞, II_f, II_∞, or III [6, Corollary 7.6]. The indecomposable, regular, right self-injective rings of Types I_f and I_∞ are easy enough to describe, since those of Type I_f are the simple artinian rings, while those of Type I_∞ are the endomorphism rings of infinite-dimensional right vector spaces [6, Theorem 5.4].

In the remaining cases, however, very little is known. The center suggests itself as a reasonable invariant with which to distinguish among different rings of the same type, particularly in the indecomposable case, where the center is a field. In this paper, we develop some techniques for calculating centers, and we apply these techniques to the standard Type II examples (described below). In particular, we show that any field can be the center of a simple, regular, right and left self-injective ring of Type II, and that the standard Type II examples can be distinguished by means of their centers.

Both of the standard Type II examples are built up from certain sequences of simple artinian rings, one by the completion of a direct limit, the other by a factor ring of the direct product. The second of these is the easiest to describe, as follows. Let $R_1, R_2, \cdots$ be simple artinian rings whose composition series lengths are unbounded, and set $R = \Pi R_n$. If M is any maximal two-sided ideal of R which contains $\bigoplus R_n$, then it follows from [6, Corollary