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RANDOM FIXED POINT THEOREMS FOR MULTIVALUED MAPPINGS

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We give some random fixed point theorems for random operators which are defined on subsets of a separable Banach space and whose values are subsets of the Banach space. The domains are allowed to be random. One of the results is a stochastic version of the Bohnenblust-Karlin-Kakutani fixed point theorem for set-valued maps.

1. Introduction. The Prague school of probabilists in the Fifties introduced the study of random fixed point theorems (cf. e.g., [10]). Recently the interest in these questions has been revived, especially by the review article [3]. Answers to some of the research problems mentioned there have been given in [5], [6], [7]. In this paper we will answer the research problem asking for a stochastic version of the Bohnenblust-Karlin fixed point theorem for set-valued maps ([4], cf. also [18]), which was proved for finite dimensional spaces by Kakutani.

A random fixed point theorem for another class of set-valued maps was recently proved in [13]. A good historic survey about fixed point theorems for set-valued maps can be found in [9].

2. Definitions and preliminary results. Throughout this paper, let X be a real separable Banach space, $(\Omega, \mathscr{A}, \mu)$ a σ-finite measure space. We will use the words "stochastic" and "random" interchangeably also if μ is not a probability measure. By 2^X we denote $\{A/A \subseteq X \wedge A \neq \phi \wedge A \text{ closed}\}$, by $CB(X) = \{A/A \in 2^X \wedge A \text{ bounded}\}$ and by $CC(X) = \{A/A \in 2^X \wedge A \text{ convex}\}$.

DEFINITION 1. Let $C: \Omega \to 2^X$ be a set-valued map. We call C "measurable" iff for all open $D \subseteq X$, $\{\omega \in \Omega / C(\omega) \cap D \neq \phi\} \in \mathscr{A}$. (Note that this is called "weakly measurable" in [12].) We call C "separable" iff it is measurable and there exists a countable set $Z \subseteq X$ such that for all $\omega \in \Omega$, $\text{cl}\,(Z \cap C(\omega)) = C(\omega)$. The "graph of C" is defined as $\text{Gr}\ C = \{(\omega, x) \in \Omega \times X / x \in C(\omega)\}$.

It can be easily shown that if C is measurable and has closed, convex, and solid (i.e., nonempty interior) values, then C is separable. The definition of separability implies that C has closed values.

DEFINITION 2. Let $C \subseteq X$ be closed. $T: C \to 2^X$ is called "upper semicontinuous (usc)" iff for all $x \in C$, $T(x)$ is compact and for all