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SPECTRAL SYNTHESIS IN HYPERGROUPS

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A commutative hypergroup K is, roughly speaking, a space in which the product of two elements is a probability measure. Such spaces have been studied by Dunkl, Jewett, and Spector. Examples include locally compact abelian groups and double-coset spaces. K has a Haar measure m (Spector). It is shown that for several important classes of hypergroups the structure space of $L^1(m)$ is a hypergroup $\hat{K}$. For such spaces, $L^1(m)$ is shown to be regular, in fact, super-regular, and to have good approximate units. A Wiener-Tauberian theorem is given. Points in the center of $\hat{K}$ are shown to be strong Ditkin sets. Examples (due essentially to Reiter and Naimark) show that not all points in $\hat{K}$ need be spectral sets.

1. Introduction. The purpose of this paper is to determine to what extent results for the group algebra of a locally compact abelian group carry over to commutative hypergroups. The theory of topological hypergroups was initiated by Dunkl [3], Jewett [6], and Spector [12] and has recently received a good deal of attention from harmonic analysts. Throughout the paper, K will denote a commutative locally compact hypergroup such that $K^\wedge$ is a hypergroup under pointwise operations. Being commutative, K admits a Haar measure m, as shown by Spector [13]. The convolution algebra $L^1(m) = L^1(K)$ can be identified with the pointwise algebra $A(K^\wedge)$ of Fourier transforms on $K^\wedge$. The main reference will be Jewett [6] who calls hypergroups "convos." A survey of the subject appears in [10].

In §2 we establish some basic facts about $A(K^\wedge)$. $A(K^\wedge)$ is shown to be a regular algebra of functions on $K^\wedge$; in fact, $A(K^\wedge)$ is super-regular (2.9). It is shown that $A(K^\wedge)$ has some useful approximate units. A Wiener-Tauberian theorem is given. Some results on spectral synthesis are given in §3. The main result asserts that points in the center of $K^\wedge$ are strong Ditkin sets. Several examples are discussed in §4. In particular, it is observed that, in general, points of $K^\wedge$ need not be spectral sets. It is also observed that there exists nondiscrete $K^\wedge$ such that every closed subset is a Calderón set.

1.1. As remarked above, we assume throughout that

(H_1) $K^\wedge$ is a hypergroup under pointwise multiplication.

In (3.5)-(3.13) we impose another hypothesis which we now discuss.