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EXTENSIONS OF PRO-AFFINE ALGEBRAIC GROUPS

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Our objective is to make a series of reductions for the problem of computing Ext in the category of pro-affine algebraic groups over an algebraically closed field of characteristic zero, exploiting the notions of unipotence, reductiveness, and group coverings. After examining some of the properties of Ext in a more general categorical setting, due to G. Hochschild, we discuss the multiplicative character theory for our groups and obtain several consequences of simple connectedness before proceeding to the main objective.

1. Preliminaries. Let G be a group, F a field, and $g\colon G \to F$ an F-valued function on G. For each x in G, we define the left and right translates of g by x by $(x \cdot g)(y) = g(yx)$ and $(g \cdot x)(y) = g(xy)$ for all y in G, respectively. For all x and y in G, we have $(x \cdot g) \cdot y = x \cdot (g \cdot y)$ so we denote this simply by $x \cdot g \cdot y$. We call g a representative F-valued function if the functions $x \cdot g \cdot y$ with x and y ranging over G lie in a finite dimensional space of functions. This condition may easily be shown to be equivalent to the assertion that the functions $x \cdot g$ all lie in a finite dimensional space of functions, or that the functions $g \cdot x$ do. The set $\mathscr{R}_F(G)$ of all representative F-valued functions on G has the structure of a Hopf algebra over F. The F-algebra structure of $\mathscr{R}_F(G)$ is the usual one. The comultiplication $\gamma\colon \mathscr{R}_F(G) \to \mathscr{R}_F(G) \otimes \mathscr{R}_F(G)$ sends any g as above to the unique element $\sum f_i \otimes g_i$ of $\mathscr{R}_F(G) \otimes \mathscr{R}_F(G)$ for which $\sum f_i(x)g_i(y) = g(xy)$ for all x and y in G. The antipode $\eta\colon \mathscr{R}_F(G) \to \mathscr{R}_F(G)$ sends g to the function whose value at each x in G is $g(x^{-1})$, and the counit $c\colon \mathscr{R}_F(G) \to F$ is evaluation at the identity element of G.

The pair (G, A) is called a pro-affine algebraic group over F if A is a Hopf subalgebra of $\mathscr{R}_F(G)$ which separates the points of G and has the property that every F-algebra homomorphism $A \to F$ is the evaluation at some element of G. If A is finitely generated we call (G, A) an affine algebraic group over F.

More generally, if X is a set and A is an F-algebra of F-valued functions on X, we call the pair (X, A) a pro-affine algebraic variety over F if A separates the points of X and every F-algebra homomorphism $A \to F$ is the evaluation at some element of X. If A is finitely generated, we call (X, A) an affine algebraic variety over F. This is one equivalent form of the usual notion.

If (X, A) and (Y, B) are pro-affine algebraic varieties over F,