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THE FOURIER STIELTJES ALGEBRA OF A TOPOLOGICAL SEMIGROUP WITH INVOLUTION

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Let S be a topological semigroup with a continuous involution. We study a subalgebra $F(S)$ of the algebra of continuous weakly almost periodic functions on S. $F(S)$ is translation invariant, closed under conjugation and contains constants. When S has an identity, then $F(S)$ is the linear span of the cone of continuous positive definite functions on S. We show that there exists a norm $||\cdot||_\Omega$ on $F(S)$ such that $(F(S), ||\cdot||_\Omega)$ is a commutative Banach algebra which can be identified with the predual of a W^*-algebra $W^*(S)$. When S is a locally compact group, then $F(S)$ is precisely the Fourier Stieltjes algebra of S. We also show that $\sigma(F(S))$, the spectrum of $F(S)$, is a *-semigroup in $W^*(S)$, and study the relation of $\sigma(F(S_1))$ and $\sigma(F(S_2))$ when $F(S_1)$ and $F(S_2)$ are isometric isomorphic Banach algebras.

1. Introduction. Recently, Dunkl and Ramirez [5] defined a subalgebra $R(S)$ of the algebra $WAP(S)$ of complex-valued continuous weakly almost periodic functions on S. The algebra $R(S)$, called the *representation algebra* of S, is constructed by considering continuous representations of S into the unit ball of $L_\infty(X, \mu)$ with the weak*-topology, where (X, μ) is some probability measure space. They showed that $R(S)$ is translation invariant, closed under conjugation and contains all bounded continuous semi-characters on S. Furthermore $R(S)$, with an appropriate norm, becomes a commutative Banach algebra and the dual of $R(S)$ can be identified with a weak*-closed subalgebra of a commutative W^*-algebra. If G is a commutative locally compact group, then $R(G) = M(\hat{G})^\wedge$, the Fourier Stieltjes transform of the measure algebra on the dual group $\hat{G}$ (see [6, p. 80]).

Our present work deals with the study of the subalgebra $F(S)$ of $WAP(S)$ of a topological *-semigroup S (i.e., a topological semigroup with a continuous involution). If S has an identity, then $F(S)$ is the linear span of continuous positive definite function on S. Also if S is a commutative, then $F(S)$ is contained in the representation algebra $R(S)$. We show that $F(S)$ can be identified with the predual of a W^*-algebra, $W^*(S)$. Furthermore $F(S)$ with the predual norm is a commutative Banach algebra, called the Fourier Stieltjes algebra of S. The algebra $F(S)$ is also translation invariant, closed under conjugation and contains all continuous *-semi-characters of