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MINIMAL (G, τ)-EXTENSIONS

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In this paper, we are concerned with lifting minimality and topological transitivity through skew-extensions—the fibres being a compact group and the action intertwines with a group automorphism. It is shown that in the class of cocycles respecting the automorphism, these properties can be lifted when the automorphism is distal. This is obtained by a dynamical decomposition of an automorphism on a group, and subsequent analysis based on this decomposition. The lifting fails for hyperbolic automorphisms on a torus.

1. Introduction. Suppose (X, ψ) is a free abelian group extension of a minimal flow (Y, η). Then it was shown in [2] and, via different techniques, in [8] that under mild assumptions, almost all cocycle perturbations of (X, ψ) over (Y, η) are minimal. In this paper we study the corresponding problem in the more general situation when (X, ψ) is a free (G, τ)-extension of (Y, η) (see § 2 for definitions). The major dynamical results (Theorem 3.13 and Corollary 3.14) state that in cases which include finite or countably infinite dimensional tori, almost all cocycles lift topological transitivity, and, when (G, τ) is distal, they lift minimality.

In preparation for these results, detailed information on the dynamical properties of group automorphisms of compact abelian groups is necessary, and we carry out this analysis in § 2. To this end, we use a particular inverse limit decomposition of (G, τ) which identifies a distal tower and an ergodic extension. Certain aspects of this decomposition were previously studied by Seethoff and Brown, see [1]. Our results in § 2, which might be of independent interest, are the identification of the maximal equicontinuous factor in this case, and the fact that on an n-torus, distality is equivalent to some power being unipotent. Finally some indications of extensions to more general actions are given.

In order to show the main results, a notion of admissibility is required, and we discuss which (G, τ) are admissible in § 3. The remainder of § 3 is devoted to proving these results and noting some examples to illustrate the theory. One of the examples (Example 3.19) shows that even in the case of a periodic automorphism, the major result cannot be deduced from Ellis' original result.

The proof of Theorem 3.13 is a modification of Ellis' proof in [2] and we acknowledge our indebtedness to that paper.