PACIFIC JOURNAL OF MATHEMATICS
Vol. 77, No. 1, 1978

KERNEL DILATION IN REPRODUCING KERNEL HILBERT SPACE AND ITS APPLICATION TO MOMENT PROBLEMS

GARY FAULKNER AND R. SHONKWILER

We give a reformulation of Nagy's Principal Theorem in terms of a dilation of a family of operators in reproducing kernel Hilbert space. In this setting we are able to generalize Nagy's result to obtain dilations $\tilde{K}$ of reproducing kernels K derived from certain families of operators. We define the concept of positive type for kernels K whose values are unbounded operators on a Hilbert space. The construction of $\tilde{K}$ is such that it possesses a property, which we call splitting, not enjoyed by K. We show that the splitting property constitutes the utility of dilation theory and use it to solve moment problems.

Our main result in this work generalizes Nagy's Principal Theorem (cf. Nagy [14]) the central conclusion of which we give here for reference.

THEOREM. *Let Γ be a *-semi-group with identity ε and suppose $\{T_\gamma\}_{\gamma\in\Gamma}$ is a family of bounded liner transformations in the complex Hilbert space H satisfying:* (a) $T_\varepsilon = I$, (b) *T_γ considered as a function of γ is of positive type, and* (c) *T_γ is completely admissible, i.e., the following inequality obtains for all finite sums*

$$(0.1)\qquad \sum_{i=1}^{n}\sum_{j=1}^{n}\langle T_{\gamma_j^*\alpha^*\alpha\gamma_i}x_i, x_j\rangle \leqq M_\alpha^2\sum_{i=1}^{n}\sum_{j=1}^{n}\langle T_{\gamma_j^*\gamma_i}x_i, x_j\rangle$$

with constant $M_\alpha > 0$. Then there exists a representation $\{D_\gamma\}_{\gamma\in\Gamma}$ of Γ in an extension space H such that

$$T_\gamma = \tilde{P}D_{\gamma|H}\ .$$

Furthermore $\tilde{H}$ may be chosen minimal in the sense that it is spanned by elements of the form $D_\gamma x$ where $x\in H$ and $\gamma\in\Gamma$.

Recently P. Masani [15] and independently F. H. Szafraniec [16] have been able to substantially weaken Nagy's condition (0.1). The weakened versions of (0.1) are:

$$(0.2)\ (\mathrm{S})\qquad |T(\alpha)| \leqq C\cdot p(\alpha)\ ,\qquad \text{where}\quad 0 \leqq p(\alpha\beta) \leqq p(\alpha)p(\beta)\ ,$$

$$(0.2)\ (\mathrm{M})\qquad 0 \leqq T_{\gamma^*\alpha^*\alpha\gamma} < M_\alpha^2\cdot T_{\gamma^*\gamma}\ .$$

These can be shown to be equivalent.