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SUMMABILITY OF MATRIX TRANSFORMS OF STRETCHINGS AND SUBSEQUENCES

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It is well known that if a regular matrix sums every subsequence of a sequence x, then x converges. It follows trivially from this result and row finiteness of the Cesáro summability matrix that if A is a regular matrix such that Ay is Cesáro summable for every subsequence y of x, then x is convergent (not merely Cesáro summable). The purpose of the present paper is to give some general results of this type involving matrix methods that are not necessarily row finite. For example, it is shown that if T is any regular matrix summability method and A is a regular matrix such that Ay is absolutely T-summable for every stretching y of x, then x is absolutely convergent. This is done without assuming that x is bounded, and consequently, without the benefit of associativity.

The well known result mentioned above is due to R. C. Buck [2], and the trivial consequence involving the Cesàro summability matrix $(C, 1)$ can be seen as follows. If A is regular and Ay is Cesàro summable for every subsequence y of x, then $(C, 1)A$ is a regular matrix which sums every subsequence of x, since row finiteness of $(C, 1)$ gives the associativity relation $(C, 1)(Ay) = [(C, 1)A]y$. Consequently by Buck's theorem, x is convergent.

When we say that a matrix A is semiregular, we will mean that A is regular over the set of all convergent sequences of 0's and 1's. Thus $A = (a_{pq})$ is semiregular iff A satisfies the first two of the following three conditions for regularity:

1) $a_{pq} \to 0$ as $p \to \infty$, $q = 1, 2, 3, \cdots$,
2) $\sum_{q=1}^{\infty} a_{pq} \to 1$ as $p \to \infty$,
3) $\sum_{q=1}^{\infty} |a_{pq}| < K_A$, $\quad p = 1, 2, 3, \cdots$.

If ε is a positive term null sequence and each of x and y is a complex sequence, then the statement that y contains an ε-copy of x means that y contains a subsequence $\{y_{n_p}\}$ such that $|y_{n_p} - x_p| < \varepsilon_p$, $p = 1, 2, 3, \cdots$.

THEOREM 1. *If $T = (t_{pq})$ is a matrix such that $\sum_{q=1}^{\infty} |t_{pq}| < L_p$, $p = 1, 2, 3, \cdots$, A is a regular matrix, and Ay is T-summable for every subsequence y of x, then either x converges or TA is a Schur matrix,* i.e., *TA sums every bounded sequence.*

Proof. Suppose x is unbounded. Clearly A is row finite since