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# A NEW FAMILY OF PARTITION IDENTITIES

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**The partition function $A(n; k)$ is the number of partitions of $n$ with minimal difference $k$. Our principal result is that for all $k \geqq 1$, $A(n; k) \equiv B(n; k)$, where $B(n; k)$ is the number of partitions of $n$ into distinct parts such that for $1 \leqq i \leqq k$, the smallest part $\equiv i \pmod k$ is $> k \sum_{j=1}^{i-1} r(j)$, where $r(j)$ is the number of parts $\equiv j \pmod k$. This arises as a corollary to a more general result.**

The particular case $A(n; 2) = B(n; 2)$ was recently proved by Andrews and Askey [1]. It is known from the Rogers-Ramanujan identities (e.g., Harby and Wright [2], p. 291) that $A(n; 2)$ is equal to the number of partition of $n$ into parts $\equiv \pm 1 \pmod 5$. Andrews and Askey discovered a $q$-series identity due to Rogers which has the partition theoretic interpretation: $B(n; 2)$ is equal to the number of partitions of $n$ into parts $\equiv \pm 1 \pmod 5$.

The general identity. Given $k \geqq 1$, let $q(1), q(2), \cdots, q(k)$ be any complete residue system mod $k$. We define the following partition functions:

$A(n; k; q(1), \cdots, q(k); r(1), \cdots, r(k))$ = number of partitions of $n$ with minimal difference $k$ and such that for $1 \leqq i \leqq k$, there are $r(i)$ parts $\equiv q(i) \pmod k$.

$B(n; k; q(1), \cdots, q(k); r(1), \cdots, r(k))$ = number of partitions of $n$ into distinct parts such that for $1 \leqq i \leqq k$, there are $r(i)$ parts $\equiv q(i) \pmod k$, and the smallest part $\equiv q(i) \pmod k$ is $> k \sum_{j=1}^{i-1} r(j)$.

$C(n; k; q(1), \cdots, q(k); r(1), \cdots, r(k))$ = number of partitions of $n$ such that for $1 \leqq i \leqq k$, there are $r(i)$ parts $\equiv q(i) \pmod k$.

Given $r(1), \cdots, r(k)$, we set $S = \sum_{i=1}^{k} r(i)$ = number of parts in the partition.

LEMMA 1.

$$A(n; k; q(1), \cdots, q(k); r(1), \cdots, r(k)) \\ = C(n - kS(S-1)/2; k; q(1), \cdots, q(k); r(1), \cdots, r(k)) .$$

*Proof*. Given a partition of $n$ with minimal difference $k$ and $r(i)$ parts $\equiv q(i) \pmod k$, subtract $k$ from the second smallest part, $2k$ from the third smallest part, and, in general $k(j-1)$ from the $j$th smallest part. This gives us a partition of $n - kS(S-1)/2$ with $r(i)$ parts $\equiv q(i) \pmod k$ for all $i$, $1 \leqq i \leqq k$.