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SETS WITH $(d-2)$-DIMENSIONAL KERNELS

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This work is about the dimension of the kernel of a starshaped set, and the following result is obtained: Let S be a subset of a linear topological space, where S has dimension at least $d \geqq 2$. Assume that for every $(d+1)$-member subset T of S there corresponds a collection of $(d-2)$-dimensional convex sets $\{K_T\}$ such that every point of T sees each K_T via S, $(\text{aff } K_T) \cap S = K_T$, and distinct pairs aff K_T either are disjoint or lie in a d-flat containing T. Furthermore, assume that when T is affinely independent, then the corresponding set K_T is exactly the kernel of T relative to S. Then S is starshaped and the kernel of S is $(d-2)$-dimensional.

We begin with some preliminary definitions: Let S be a subset of a linear topological space, S having dimension at least $d \geqq 2$. For points x, y in S, we say x sees y via S if and only if the corresponding segment $[x, y]$ lies in S. Similarly, for $T \subseteq S$, we say x sees T (and T sees x) via S if and only if x sees each point of T via S. The set of points of S seen by T is called the kernel of T relative to S and is denoted $\ker_S T$. Finally, if $\ker_S S = \ker S$ is not empty, then S is said to be starshaped.

This paper continues a study in [1] concerning sets having $(d-2)$-dimensional kernels. Foland and Marr [2] have proved that a set S will have a zero-dimensional kernel provided S contains a noncollinear triple and every three noncollinear members of S see via S a unique common point. In [1], an analogue of this result is obtained for subsets S of R^d having $(d-2)$-dimensional kernels. Here it is proved that, with suitable hypothesis, these results may be extended to include subsets S of an arbitrary linear topological space.

As in [1], the following terminology will be used: Conv S, aff S, cl S, bdry S, rel int S and ker S will denote the convex hull, affine hull, closure, boundary, relative interior and kernel, respectively, of the set S. If S is convex, dim S will represent the dimension of S.

2. Proof of the theorem.

THEOREM. *Let S be a subset of a linear topological space, where S has dimension at least $d \geqq 2$. Assume that for every $(d+1)$-member subset T of S there corresponds a collection of $(d-2)$-dimen-*