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# CHEBYSHEV CENTERS AND UNIFORM CONVEXITY

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**If $E$ is a uniformly convex Banach space and $T$ is any topological space, then in the space $X = C(T, E)$ of $E$-valued bounded continuous functions on $E$, every bounded set has a Chevyshev center. Moreover, the set function $A \to Z(A)$, corresponding to $A$ the set of its Chebyshev centers, is uniformly continuous on bounded subsets of the space $\mathscr{B}(X)$ of bounded subsets of $X$ with the Hausdorff metric. This is contrasted with the fact that a normed space $X$ in which $Z(A)$ is a singleton for every bounded $A$ is uniformly convex iff $A \to Z(A)$ is uniformly continuous on bounded subsets of $\mathscr{B}(X)$.**

Let $(X, d)$ be a metric space. Denoto by $\mathscr{B}(X)$ the space of nonempty bounded subsets of $X$ and let $h$ be the Hausdorff semimetric on $\mathscr{B}(X)$:

$$h(A, B) = \max(\sup_{u\in A} \inf_{v\in B} d(u, v), \sup_{v\in B} \inf_{u\in A} d(u, v)) .$$

For $x \in X$, $r \geqq 0$, let $B(x, r) = \{y \in X; d(x, y) \leqq r\}$ be the closed $r$-ball around $x$. For $A \in \mathscr{B}(X)$ and $x \in X$ denote $r(x, A) = \inf\{r \geqq 0; B(x, r) \supset A\}$, $r(A) = \inf_{x\in X} r(x, A)$ is the *Chebyshev radius of* $A$, and $Z(A) = \{x \in X; r(x, A) = r(A)\}$ is the set of *Chebyshev centers of* $A$. For $Y \subset X$ we can consider also the *relative Chebyshev radius* of $A$ in $Y$, $r_Y(A) = \inf_{y\in Y} r(y, A)$, and the set of *relative Chebyshev centers of $A$ in $Y$*, $Z_Y(A) = \{y \in Y; r(y, A) = r_Y(A)\}$. In the case that $A = \{x\}$ is a singleton, then $Z_Y(A)$ is just the set of best approximations in $Y$ to $x$, $P_Y x$.

We say that $X$ *admits centers* if every bounded set in $X$ has Chebyshev centers. The classical Banach spaces, i.e., the spaces $L_p(\mu)$, $1 \leqq p \leqq \infty$, over any measure space and the spaces $C(T)$ of continuous real-valued functions on compact Hausdorff $T$, admit centers ([1], [3]). However, Garkavi ([1]) gave an example of a 3-point set in a maximal subspace $H$ of $C[0, 1]$ which has no Chebyshev center in $H$. The problem of characterizing all Banach space which admit centers is still open.

Ward ([5]) proved that the space $C(T, E)$ of $E$-valued bounded continuous functions on the topological space $T$, with the norm $||x|| = \sup_{t\in T} ||x(t)||$, admits centers in each of the following two cases: (a) $E$ is a finite-dimensional strictly convex (hence uniformly convex) normed space and $T$ is paracompact. (b) $E$ is a Hilbert