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ON THE OSCILLATORY AND ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF FIFTH ORDER SELFADJOINT DIFFERENTIAL EQUATIONS

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In this paper the fifth order selfadjoint differential equation

$$(1) \qquad (z''' + 2p(x)z)' + 2p(x)z' = 0$$

is considered under the assumption that $p(x)$ is a positive continuous function defined on the half axis $[0, \infty)$. The oscillation and asymptotic properties of certain solutions of (1) will be discussed after which connections between the solutions of (1) and the solutions of the fourth order differential equation

$$(2) \qquad y'''' - p(x)y = 0$$

are investigated. More spcifically, it is shown that (1) is oscillatory if and only if (2) is oscillatory.

While the literature is unusually scanty on the solutions of odd order selfadjoint differential equations, oscillation properties of the selfadjoint third order equation

$$(3) \qquad (y'' + 2b(x)y)' + 2b(x)y' = 0$$

has been studied by several authors, including J. H. Barrett [3], G. D, Jones [4] and S. C. Tefteller [7]. All of these works utilized the fact that if u and v are solutions of

$$(4) \qquad w'' + b(x)w = 0\ ,$$

then the functions u^2, uv, and v^2 are solutions of (3).

Recall that a nontrivial solution of (1) or (2) is said to be *oscillatory* if it has arbitrarily large zeros, otherwise it is termed *nonoscillatory*. In case (1) has an oscillatory solution we say that (1) is *oscillatory*. A similar definition holds for equation (2). For convenience, the term "solution" for the remainder of this work will refer to nontrivial solutions unless otherwise noted.

Tefteller in [7] proved that (3) is oscillatory if and only if (4) is oscillatory. While in [4] it was shown that if (3) is oscillatory then the solution space of (3) has a basis with i oscillatory solutions and $3 - i$ nonoscillatory solutions for $i = 0, 1, 2, 3$. It is these observations and the aforementioned connections between the solutions of (1) and (2) that motivates this study. Finally, we refer to the