PACIFIC JOURNAL OF MATHEMATICS
Vol. 77, No. 2, 1978

THE STRONG BIDUAL OF $\Gamma(K)$

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Let A be a C^*-algebra, K the Pedersen ideal of A, and $\Gamma(K)$ the two-sided multipliers of K under the κ-topology. In this paper a study is made of the strong bidual of $\Gamma(K)$, which we denote by $\Gamma(K)''$. Here it is shown that the Arens products in $\Gamma(K)''$ are well defined and coincide, and therefore make $\Gamma(K)''$ a *-algebra; moreover, if A is a PCS-algebra or has a σ-compact spectrum, it is shown that $\Gamma(K)''$ is a metrizable b^*-algebra which is isometrically *-isomorphic to the Cartesian product of W^*-algebras.

Now suppose A is just a Banach algebra. In [3] Arens defined two natural extensions of the product of A to the strong bidual A''. If it is assumed that A is also a *-algebra, then it is well known and easy to verify that a natural extension of the involution of A can be defined for A'' whenever the two products coincide. When A is a C^*-algebra, it is also well known that the two Arens products for A'' coincide and the resulting *-algebra is a C^*-algebra. For proofs of these facts, we refer the reader to [7], [20], and [23].

The fact that A'' is a C^*-algebra whenever A is a C^*-algebra has been very useful. For example, by focusing on certain elements from A'', noncommutative analogues of the bounded Baire, Borel, and semicontinuous functions on a locally compact Hausdorff space have been developed. All of this suggests that the strong bidual of other locally convex topological algebras may be of equal importance. It is the purpose of this paper to study the strong bidual of an important class of topological algebras which we will shortly define.

Let A be a C^*-algebra, K the Pedersen ideal of A, $\Gamma(K)$ the two-sided multipliers of K under the κ-topology. When A is commutative, $\Gamma(K)$ is the *-algebra of all complex valued continuous functions defined on the spectrum of A under the compact open topology. In the general case, $\Gamma(K)$ can be viewed as a *-algebra of unbounded operators on a pre-Hilbert space that has many of the nice properties of C^*-algebras. In this paper we shall study the strong bidual of $\Gamma(K)$, which we denote by $\Gamma(K)''$. We show that the Arens products in $\Gamma(K)''$ are well defined and coincide, and therefore make $\Gamma(K)''$ a *-algebra. Moreover, if A is a PCS-algebra or has a σ-compact spectrum, we show that $\Gamma(K)''$ is a metrizable b^*-algebra which is isometrically *-isomorphic to the Cartesian product of W^*-algebras. It is our opinion that noncommutative ana-