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ON THE GEOMETRY OF COMBINATORIAL MANIFOLDS

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On a smooth manifold there are classical relations between vector fields and derivations of the smooth function algebra, and between differential forms and alternating linear maps of vector field tuples. In this paper similar relations are obtained for combinatorial manifolds. As an application of these results the existence of connexions and parallel translation on combinatorial manifolds is established.

0. Introduction. The basic theme of (6) and (7) is that there is a striking similarity between the geometry of smoth manifolds and the geometry of simplicial complexes. The purpose of this paper is to continue this theme for smooth manifolds and combinatorial manifolds. (Note: Henceforth a *combinatorial n-manifold M* is the geometric realization of a simplicial complex for which the closed star of each point can be mapped by a homeomorphism onto a combinatorial n-ball in R^n in such a manner that each simplex of M is mapped affinely to a simplex in R^n. Furthermore all combinatorial manifolds are assumed to have no boundary. See (1) and (9) for related definitions.)

Section 1 is devoted to a brief review of some of the terminology and results of (6) and (7). The goal of §2 is the characterization of continuous vector fields on combinatorial manifolds. The main technical results of this paper are proved in §3; these results are complied in the following statement.

THEOREM. *Let M be a combinatorial n-manifold, $A(M)$ the ring of piecewise smooth real-valued functions on M, $\mathscr{X}(M)$ the $A(M)$-module of continuous vector fields on M, and $E(M)$ the $A(M)$-module of piecewise smooth 1-forms on M. Then*:

(1) *there is an $A(M)$-module isomorphism between $\mathscr{X}(M)$ and the module $\mathscr{D}(M)$ of derivations of $A(M)$; consequently $\mathscr{X}(M)$ is a Lie algebra over R with respect to*

$$[X, Y]f = X(Yf) - Y(Xf)$$

for $X, Y \in \mathscr{X}(M)$ and $f \in A(M)$,

(2) *there is an $A(M)$-module isomorphism between $E(M)$ and the module* $\mathrm{Hom}_{A(M)}(\mathscr{X}(M), A(M))$ *of $A(M)$-linear maps from $\mathscr{X}(M)$ to $A(M)$,*

(3) *there is an $A(M)$-module isomorphism between $\Lambda^q E(M)$*