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A VANISHING THEOREM FOR THE MOD p MASSEY-PETERSON SPECTRAL SEQUENCE

MASAMITSU MORI

A vanishing theorem and periodicity theorem for the classical mod 2 Adams spectral sequence were originally proved by Adams [1]. The results were extended to the unstable range by Bousfield [2]. The purpose of this paper is to show the analogue of Bousfield's work for the mod p unstable Adams spectral sequence of Massey-Peterson type (called the mod p Massey-Peterson spectral sequence), where p is an odd prime. The results generalized those obtained by Liulevicius [5], [6] to the unstable range. As an immediate topological application we have the estimation of the upper bounds of the orders of elements in the p-primary component of the homotopy groups of, for example, an odd dimensional sphere, Stiefel manifold, or H-space.

1. The vanishing theorem. Let A denote the mod p Steenrod algebra. Let $A\mathscr{M}$ the category of unstable left A-modules and $\mathscr{M}A$ thecategory of unstable right A-modules. We may define $\mathrm{Ext}^s_{A\mathscr{M}}$, $s \geqq 0$, as the sth right derived functor of $\mathrm{Hom}_{A\mathscr{M}}$, and similarly define $\mathrm{Ext}^s_{\mathscr{M}A}$, since $A\mathscr{M}$ and $\mathscr{M}A$ are abelian categoriesw ith enough projectives. Note that, if $M \in A\mathscr{M}$ is of finite type, then

$$\mathrm{Ext}_{A\mathscr{M}}(M, Z_p) = \mathrm{Ext}_{\mathscr{M}A}(Z_p, M^*) .$$

Recall the mod p Massey-Peterson spectral sequence (see, for example, [4]). Let X be a simply connected space with $\pi_*(X)$ of finite type. Suppose that $H^*(X; Z_p) \cong U(M)$, $M \in A\mathscr{M}$, where $U(M)$ is the free unstable A-algebra generated by M. Then there is a spectral sequence $\{E_r(X)\}$ with

$$d_r\colon E_r^{s,t}(X) \longrightarrow E_r^{s+r,t+r-1}(X) ,$$

such that

$$E_2^{s,t}(X) \cong \mathrm{Ext}^{s,t}_{A\mathscr{M}}(M, Z_p) ,$$

and

$$E_\infty(X) \cong \mathrm{Gr}\, \pi_*(X)/(\text{torsion prime to } p) .$$

Let Λ be the bigraded differential algebra over Z_p introduced by Bousfield et al [3], which has multiplicative generators λ_i of