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MINIMAL SPLITTING FIELDS FOR GROUP REPRESENTATIONS, II

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Let p be an arbitrary prime and m an arbitrary positive integer. A finite group G is constructed which has an irreducible complex representation T with character χ such that the Schur index of χ over Q is p but the minimum of $[K: Q(\chi)]$, taken over all abelian extensions K of Q in which T is realizable, is p^m.

Let Q denote the rationals and, for n a pasitive integer, let ε_n denote a primitive nth root of unity over Q. Let χ be the character afforded by a complex irreducible representation T of a finite group G of order n and let $m_Q(\chi)$ denote the Schur index of χ over Q. In view of the famous theorem of R. Brauer that T is realizable in $Q(\varepsilon_n)$, it is natural to ask how close to $m_Q(\chi)$ is $\min[L: Q(\chi)]$, where the minimum is taken over all subfields L of cyclotomic extensions of Q in which T is realizable. Our main result shows that the above minimum is not, in general, very close to $m_Q(\chi)$.

THEOREM 1. *Let p be an arbitrary prime and m an arbitrary positive integer. Then there exists a finite group G of exponent n and an irreducible complex representation T of G affording the character χ such that $m_Q(\chi) = p$ and $p^m = \min[L: Q(\chi)]$ where the minimum is taken over all abelian extensions L of Q in which T is realizable. The minimum is attained at a subfield of $Q(\varepsilon_n)$.*

There are several results in the recent literature that are similar in spirit to the above theorem. In [5], Schacher produces an example of a finite dimensional division algebra D with center an abelian extension of Q with the property that no maximal subfield of D is an abelian extension of Q. It can be shown, however, that his example does not arise from a group algebra of a finite group. Given an arbitrary prime p and an arbitrary integer $m \geqq 2$, Ford and Janusz in [3] produce an example of a complex irreducible representation T with character χ of a finite group G such that $m_Q(\chi) = p$, $\varepsilon_{p^2} \notin Q(\chi)$, and for some $r > m$, T is realizable in $Q(\chi)(\varepsilon_{p^r})$ but not in any proper subfield. It can be shown, however, that T is also realizable in a subfield L of $Q(\varepsilon_n)$, n the exponent of G, where $[L: Q(\chi)] = p$. In [2] an example is found of an irreducible complex representation T with character χ of a finite group G of order n with the property that T is not realizable in any subfield L of $Q(\varepsilon_n)$